A'LEVEL PHYSICS NOTES

P510/1

MECHANICS

DIMENSIONS

Fundamental quantities are those physical quantities which cannot be expressed in terms of any other quantities using any mathematical equation e.g. mass (M), length (L) and time (T).

Quantity	S.I Unit	Symbol of S.I Unit
Mass	Kilograms	kg
Time	seconds	S
Length	metres	M
Temperature	Kelvin	K
Current	Ampere	A

Derived quantities

These are quantities which can be expressed in terms of the fundamental quantities e.g.

Area – (length) ^{2}

Volume = (length) ^{3}

Density = $\dfrac{mass}{(length)^3}$

Velocity = $\dfrac{length}{time} = ms^{-1}$

Acceleration = $\dfrac{length}{(time)^2}$

Exercise 1:

Express the following derived quantities in terms of the fundamental quantities;

(a) Force (b) Pressure (c) work (d) momentum

Dimensions of a physical quantity

Is the way the fundamental quantities of a derived quantity are related or are the powers to which the fundamental quantities are raised in derived quantity.

Symbol of dimension [ ]

e.g. [Mass] -This means that the dimensions of mass.

[Mass]= M

[Time] = T

[Length] = L

[Area] = L^2 -This means that the dimension of area is 2-in length.

[Volume] = L^3 [Force] = MLT^{-2}

[Density] = ML^{-3} [Pressure] = $MT^{-2}L^{-1}$

[Velocity] =LT^{-1} [Work] = ML^2T^{-2}

[Acceleration] =LT^{-2} [Power] = ML^2T^{-3}

Exercise 2:

Find the dimensions of the following derived quantities in terms of M, L, and T;

(a) Density (b) pressure (c) power (d) momentum

Quantities without units are called dimensionless quantities e.g.

(i) Relative density (iv) Mechanical advantage

(ii) Refractive index (v) Natural numbers

(iii) Geometrical ratios

Exercise 3:

Which of the following quantities are dimensionless quantities?

Weight, velocity ratio, logarithmic numbers, energy, efficiency, coefficient of friction,

Application of dimensions

(i) Checking for the correctness of the equation.

An equation is correct when it is dimensionally consistent i.e. when dimension on the left hand side (L.H.S) are equal to dimensions on the right hand side (R.H.S.)

Example:

Prove that the following equations are dimensionally consistent

(i) $F = \frac{mv^2}{r}$ Where F = force, m =mass, V = velocity, r = radius

[L.H.S] = [F] = MLT^{-2}

[R.H.S] = $\frac{[M].[V]^2}{[r]} = \frac{M\left(LT^{-1}\right)^2}{L} = \frac{ML^2T^{-2}}{L} \quad = MLT^{-2}$

Since [L.H.S] = [R.H.S], then the equation is dimensionally consistent!

(ii) S=ut+½at ^{2}

[L.H.S] = [S] = L

[R.H.S] = [ut+½at^2] = [U] [t] +½ [a] [t^2]

$$\frac{L}{T}xT + ½\text{x}\,\frac{\text{L}}{\text{T}^2}xT^2$$

$$\frac{L}{1} + \frac{L}{2}$$

$$= {}^{3}\!/_{2}\,L$$

Since [L.H.S] = [R.H.S], then the equation is dimensionally consistent!

In the above example 3/2 is just a number so it is not a dimension. You have to consider the power on L.

Note. All current equations are dimensionally consistent but not all dimensionally consistent equations are correct.

e.g.

V=u+2at.

[v] =LT^{-1}

[U+2at] = $\frac{L}{T} \times 2.\frac{L}{T^2} \times T$

$$\frac{L}{T} + \frac{2L}{T}$$

$$= 3LT^{-1}$$

Dimensionally consistent, but it is a wrong equation.

2. Derive the equation

Example 1: Given that the pressure exerted by the liquid in a container depends on:

(i) Depth (h) of the liquid

(ii) Density of the liquid (ρ)

(iii) Acceleration due to gravity(g).

Use the method of dimension to determine the expression fro pressure

Where K is a dimensionless constant.

$$p = kh^x p^y g^z$$

$$[p] = [h]^x [p]^y [g]^z$$

$$ML^{-1}T^{-2} = L^X .(ML^{-3})^Y .(LT^{-2})^Z$$

$$ML^{-1}T^{-2} = L^{X-3Y+X} .M^Y .T^{-2Z}$$

Comparing powers

<u>For M:</u>

$$M^1 = M^y$$

$$y = 1$$

<u>For T:</u>

$$T^{-2} = T^{-2Z}$$

$$-2 = -2Z$$

$$Z = 1$$

<u>For L:</u>

$$L^{-1} = L^{X-3Y+Z}$$

$$-1 = x - 3 + 1$$

$$x = 1$$

Since x *=1, y = 1, z = 1,* then

P= khpg

Example 2: Given the period of oscillation (Ψ) of a pendulum bob is according to the equation

Ψ = $kl^x.g^y.m^z$. Where *l* is the length of a pendulum, *m* is the mass of bob and *g* is the acceleration due to gravity. Find the values of *x, y and z*

Ψ = $kl^x.g^y.m^z$

[Ψ] = k $[L]^x [g]^y [m]^z$

T = k $L^x (LT^{-2})^y M^{z.}$

T = k $L^{x+y} T^{-2y} M^z$

Comparing powers

For T:

$T = T^{-2y}$, $y = -½$

For M:

$M^{0} = M^{z}$, $Z = 0$

For M:

$L^{0} = L^{x+y}$, $x + y = 0$

But $y = -½$

Hence $x = ½$

$\Psi = \underline{K L^{½} g^{-½} M^{0}}$

Note: The method of dimensions does not provide the method for finding the constant k in the above two examples!

Exercise 4:

1. Find the values of x, y and z in the equation below:

$F = \rho^{x} V^{y} a^{z}$. Where F is the force, ρ is density, V is Volume and a is acceleration due to gravity.

2. Find the values of x, y and z in the equation below:

$F = k\eta^{x} v^{y} a^{z}$. Where F is the force, **η** is coefficient of viscosity, ν is velocity and **a** is radius. $[\eta] = ML^{-1}T^{-1}$

3. Assuming the frequency (F) of a uniform stretched wire depends only on the mass per unit length (μ), the length of wire vibrating (L), the tension (T) of the stretching wire, Find the relationship between these quantities.

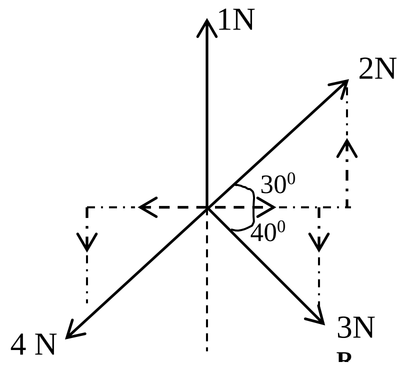
VECTOR & SCALER QUANTITIES

Vector quantities are those with both magnitude and direction, e.g. acceleration, velocity, displacement, pressure, weight.

Scalar quantities are physical quantities with only magnitude, e.g. speed, distance, time, mass.

Example

1. Find the resultant force



$F_x = 2\cos30^0 + 3\cos40^0 - 4\cos60^0$.

$F_x = 2\frac{\sqrt{3}}{2} + 3\cos 40 - 4x\frac{1}{2}$ $= 2.03N$

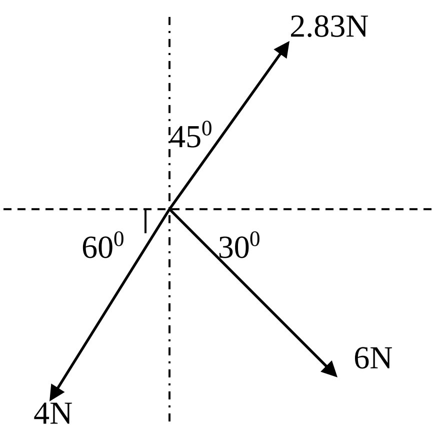
$F_y = 2\sin30^0 - 3\sin40^0 - 4\sin60^0 + 1$

$F_y = 2.\frac{1}{2} - 3\sin 40 - 4.\frac{13}{2} + 1$ $= - 3.39N$

Resultant = $\sqrt{F_x{}^2 + F_y{}^2}$ $= \sqrt{2.03^2 + 3.39^2}$

$= 3.95N$

2.



Forces of 2.83N, 4N and 6N act on a particle at Q as shown above. Find the resultant force on the particle

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2.38\cos 45 \\ 2.83\sin 45 \end{pmatrix} + \begin{pmatrix} 6\cos 30^0 \\ -6\sin 30^0 \end{pmatrix} + \begin{pmatrix} -4\cos 60^0 \\ -4\sin 60^0 \end{pmatrix}$$

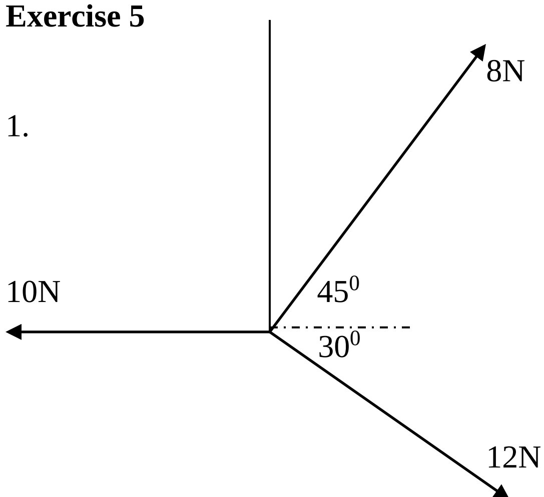
$$= \begin{pmatrix} 2 + 5.2 - 2 \\ 2 - 3 - 3.46 \end{pmatrix} = \begin{pmatrix} 5.2 \\ -4.46 \end{pmatrix}$$

$$resul\tan t = \sqrt{\ } (5.2)^2 + (4.46)^2$$

$$= 6.85N$$

Exercise 5

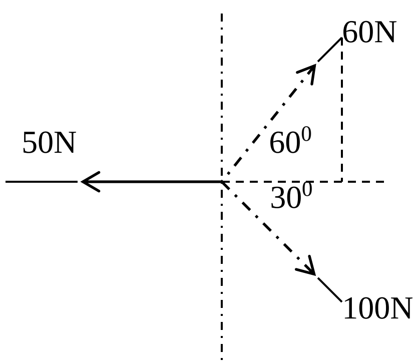
1.



Find magnitude and direction of the resultant.

2) Three forces as shown below act on a body of man 5.0kg.

find the acceleration of the body



Uniform motion in a straight line / kinematics

Displacement – distance covered in a specified direction

Speed – the rate of change of distance

Velocity – the rate of change of displacement

Acceleration – the acceleration of a moving object at an instant is the rate of change of velocity at that instant.

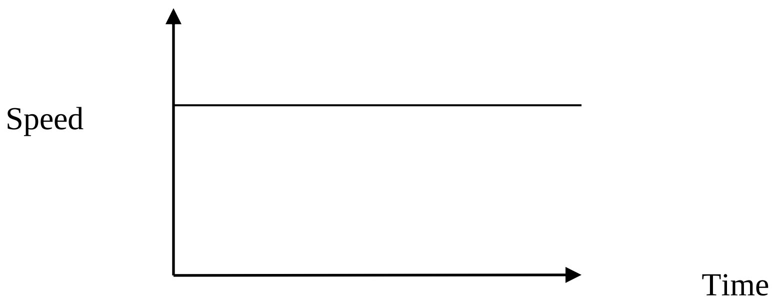
Velocity and acceleration are vector quantities where as speed and distance are scalar quantities

Uniform velocity motion

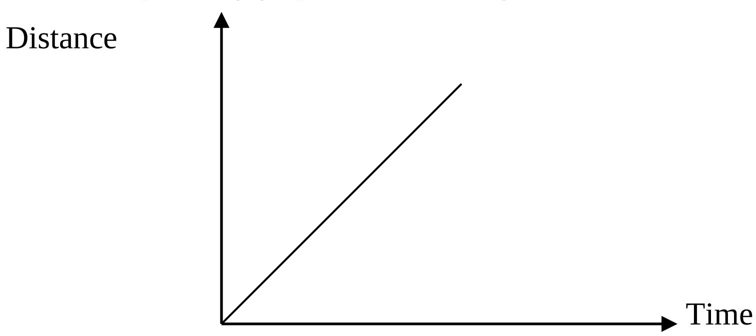
Consider a body moving in a straight line with uniform speed.

During this motion, the body undergoes equal displacement in equal successive time intervals.

The graph of speed against time has the form;



The corresponding graph of distance against time is:

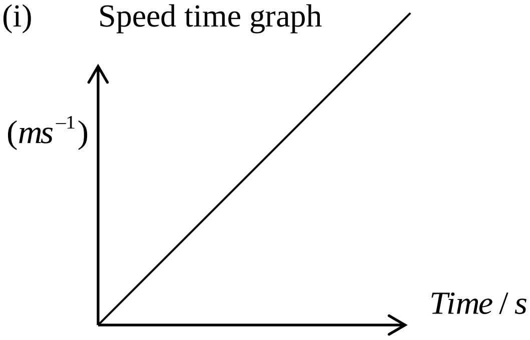


In drawing the graph, it is assumed that the body was at the origin at time t=0,

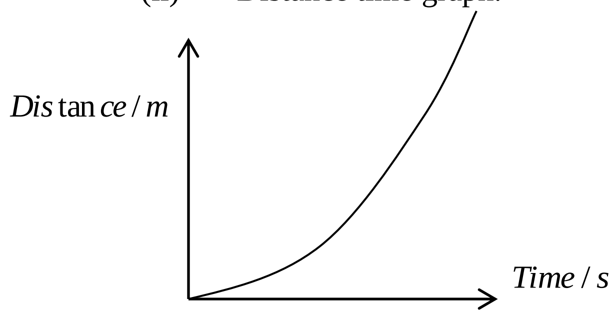
Uniformly accelerated motion

If the velocity changes by equal amounts in equal times, no matter how small the time intervals may be, the acceleration is said to be uniform.

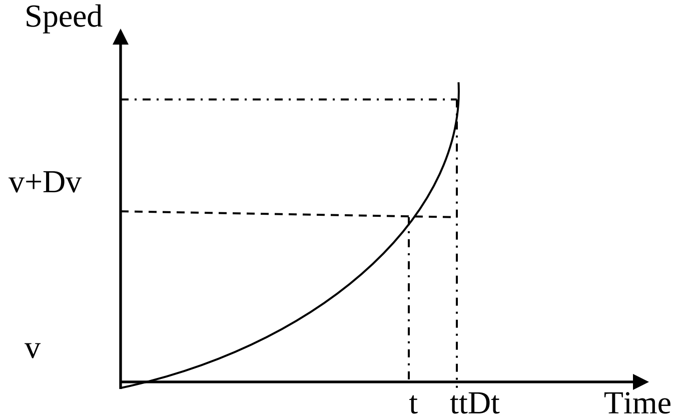
(i) Speed time graph



(ii) Distance time graph.



Suppose a body's speed varies with time . the speed Vs time are might have the form:-



The ratio $\frac{\Delta v}{\Delta t}$ is the average acceleration during the time interval

Δt it is equal to the slope chord PQ

The instantaneous acceleration at time t is

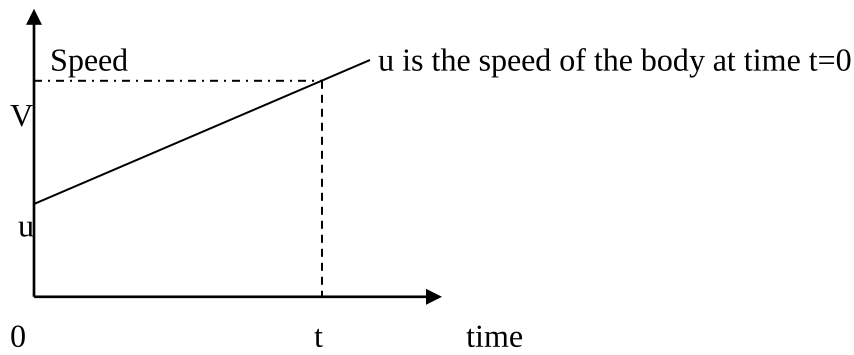
$$a = \frac{dv}{at} = lin\frac{\Delta v}{\Delta t} = \Delta t \to 0$$

Slope of the tangent at the speed-time curve at point P.

The motion of the body is said to be uniformly acceleration if a is constant.

Thus the speed against time graph for uniformly accelerated motion has the form shown

Thus the speed against time graph for uniformly accelerated motion has the form shown below

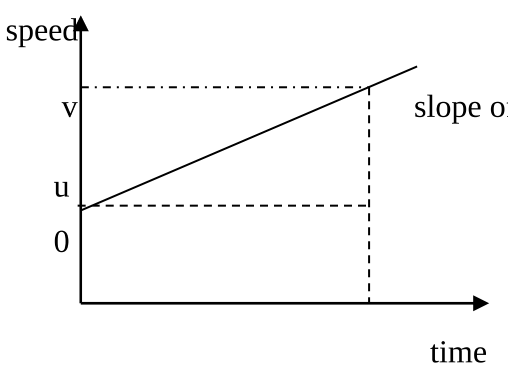


The average acceleration, a = $\frac{v-u}{t}$. In this case, the average acceleration is also the instantaneous acceleration.

$$\frac{v-u}{t} = a$$
$$v = u + at$$

Equations of uniformly accelerated motion

The graph of speed against time for uniformly accelerated motion has the form shown:-



slope of graph, a = $\frac{v-u}{t}$

Thus v=u +at…………………(i)

The distance travelled, S, in time t is got by finding the area under the speed against time graph.

S = area of the trapezium

$$\left(\frac{v+u}{2}\right)t.................(A)$$

Replacing v by equation (i) we get

$$S=\left(\frac{u+at+u}{}2\right)t$$

$$\therefore S=ut+\tfrac{1}{2}at^2.....................(2)$$

Note that this result can be got easily from the relation distance travelled

= average speed x time

$$=\left(\frac{v+u}{2}\right)t$$

Suppose we put t= $\frac{v-u}{a}$ in equation (A)

$$\left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)=\frac{v^2-u^2}{2a}$$

S = $v^2-u^2=2as$

$$\therefore V^2=u^2+2as...........................(3)$$

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$$v^2-u^2=2as$$

$$\therefore v^2=u^2+2as.............................(3)$$

Equations 1, 2, & 3 are the equations of uniformly accelerated motion.

Vertical motion under gravity: Free fall

Consider a body falling in a vacuum. Such a body is acted on by the gravitational force alone. The fall is referred to as free fall. In practice, when bodies fall in air, they are acted on by air resistance which will have significant effects on the body's motion if the body's mass is small while the surface area is large, as is the case when a piece of paper is allowed to fall in air.

The acceleration of a freely falling body is constant, and is called the acceleration due to gravity, and is denoted by *g*. It has a value of $9.81ms^{-2}$ near the poles $9.78ms^{-2}$ at the equator.

Exercise 6

1. Write down the equations of motion for a freely falling body.
2. Suppose a body is projected upwards with initial velocity *u*. Find the expressions for the time taken to reach the maximum height and also the maximum height attained.

Example:-

1. A ball is thrown vertically upwards with initial speed $20ms^{-1}$. After reaching the maximum height and on the way down it strikes a bird 10m above the ground.

a) How high does the ball rise:-

$V^2 = U^2 + 2as.$ U = $20ms^{-1}$ a = $-9.8ms^{-2}$ V = 0

$0 = (20)^2 + 2(-9.8)s$

$s = \frac{400}{19.6} = 20.4m$

b) How fast is the ball moving when it strikes the bird

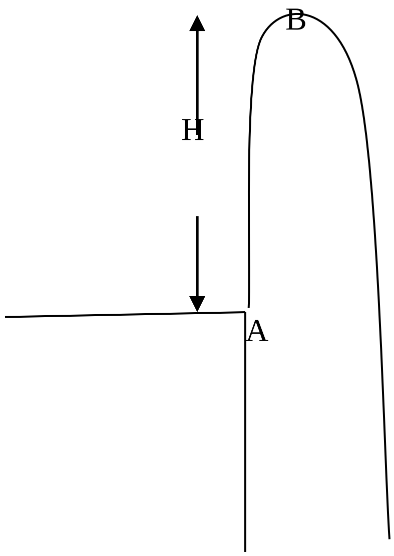
$S = (20.4 - 10) = 10.4m$

$V^2 = U^2 + 2as$

$V^2 = (0)^2 + 2(-9.8)(-10.4)$

$V = \pm 14.28ms^{-1}$

2. A stone is thrown vertically upwards with a speed of $10ms^{-1}$ from a building. If it takes 2.5 seconds to reach the ground, find the height of the building.



Between AB, u = 10ms^{-1}, a = -9.81ms^{-2} v = 0

$$v = u + at$$
$$v = 10 - 9.81t$$
$$0 = 10 - 9.81t$$
$$t = 10/9.81$$
$$= 1.02s$$

Distance AB, H = ut + ½at^2

= 10x1.02 - ½x9.81x (1.02)^2 = 5.1m

Time taken to travel distance BC = 2.5 – 1.03 = 1.48s

Distance BC = $u_b$xt + ½at^2

But u_b = 0, a = 9.81, t = 1.48s

BC = 0x1.48 + ½x9.81x (1.48)^2 = 10.7m

There the height of the building is 10.7 – 5.1 = 5.6m

Exercise 7

1. A ball is thrown straight upwards with a speed ams^{-1} from a point hm above the ground. Show that time taken to reach the ground is

$$t = \frac{u}{9}\left[1 + \left(1 + \frac{2gh}{u^2}\right)^{1/2}\right]$$

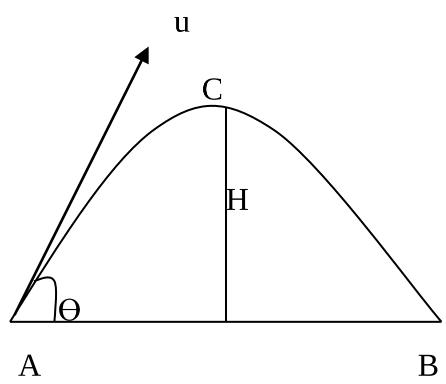
2. A motorist travelling at a constant speed of 50 kmh^1 passes a motorcyclist just starting off in the same direction. If the motorcyclists maintains a constant acceleration of 2.8ms^{-2} calculate; (i) Time taken by motorcyclist to catch up with the motorist. (9.9s)

(ii) The speed at which the motorcyclist overtakes the motorist.(27.72ms^{-1})

(iii) The distance travelled by the motorcyclists before overtaking.(137.2m)

PROJECTILE

Consider the motion of an object which is projected with a velocity u at an angle Q to the horizontal



Ө - angle of projection.

ACB followed by the object is called its trajectory.

Horizontal motion

Horizontal component of velocity is got by

$V_x = U_x + a_x t$. Where Vx, U_x and a_x are the velocity of a body at any time t, initial component of velocity and horizontal acceleration respectively.

But $U_x = U\cos\theta$, $a_x = 0$

Hence $V_x = U\cos\theta$ -------------(1)

From the above equation the horizontal velocity is constant throughout motion.

The horizontal distance travelled after time t is

$$X = u_x t + \frac{1}{2} a_x t^2$$

Where X is the horizontal distance covered by the object

But $a_x = 0$

$\therefore x = Ut\cos\theta$...............(2)

Vertical motion

$V_y = U_y + a_y t$ where V_y, U_y and a_y are the vertical velocity of a body at any time, t, initial velocity component of velocity and vertical acceleration respectively.

$U_y = U\sin\theta, a_y = -g$

$V_y = U\sin\theta - gt..................(3)$

The vertical displacement, y, is obtained below

$y = U_y t + \frac{1}{2} a_y t^2$

But $U_y = U\sin\Theta$, $a_y = -g$

Hence

$y = (U\sin\Theta)\,xt - \frac{1}{2}gt^2$……………………………………(4)

Speed, V, at any time t is given by

$V = \sqrt{(V_x^2 + V_y^2)}$ ……………………………………….(5)

The angle, α, the body makes with the horizontal after t is given by

$\tan\alpha = \frac{Vy}{Vx} = \frac{U\sin\theta - gt}{U\cos\theta}$ ……………………………………….(6)

<u>Maximum height, H</u>

At maximum height, Vy =0

$Vy^2 = U_y^2 + 2aH$

$0 = (U\sin\theta)^2 - 2ghH$

$H = \frac{U^2\sin^2\theta}{2g}$..(7)

Time to reach the maximum heights

Using V = u + at

$$0 = Uy + a_y t$$

$$0 = U\sin\theta - gt$$

$$t = U\frac{\sin\theta}{g}..(8)$$

Time of flight, T

The time taken by the projectile to move from the point of projection to a point on the plane through the point of projection where the projection lies i.e. time taken to move from A to B.

$$atB,\ \mathrm{y} = 0$$

$$\mathrm{y} = \mathrm{ut\sin}\theta - \frac{\mathrm{gt}^2}{2}$$

$$0 = 2at\sin\theta - gt^2$$

$$0 = t(2u\sin\theta - gt)$$

$$\text{either t} = 0 \text{ or } \mathrm{t} = \frac{2\mathrm{u}\sin\theta}{\mathrm{g}}$$

$$\text{Hence T} = \frac{2\mathrm{u}\sin\theta}{\mathrm{g}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Note: Time of flight is twice the time taken to reach height.

Ranges , R:

It is the distance between the point of projection and a point on the plane through the point of projection where the projectile lands i.e horizontal distance AB.

X= Ut cosΘ

When X=R, t = T= $\frac{2\mathrm{usin}\theta}{\mathrm{g}}$

$$\therefore R = u.\frac{2\mathrm{usin}\theta}{\mathrm{g}}.\cos\theta$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g} \ldots\ldots\ldots\ldots(10)$$

$$R = \frac{u^2\sin 2\theta}{g}$$

Equation of trajectory

$$t = \frac{x}{u\cos\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$y = ut\sin\theta - \frac{gt^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Substitute equation (1) into equation 2.

$$y = U.\frac{x}{U\cos\theta}.\sin\theta - \frac{g}{2}\frac{x^2}{U^2\cos^2\theta}$$

$$y = \frac{\sin\theta x}{\cos\theta} - \frac{gx^2}{2U^2\cos^2\theta}$$

$$y = (\tan\theta)x - \left(\frac{g}{2u^2Cos^2\theta}\right)x^2$$

$$\mathrm{y = xtan}\ \theta - \tfrac{1}{2}gx^2\ \frac{\sec^2\theta}{u_2}$$

The above equation is in the form y= Ax – Bx^2, where A and B are constants which is an equation of a parabola. Therefore the trajectory is a parabola.

Note: For any given initial speed, the range is maximum when sinΘ =1 or Θ = 45^0

R$_{max}$ = $\frac{U^2}{g}$ *(Prove it !!!!)*

Example

1. Prove that the time of flight T and the horizontal range R, of a projectile are connected by the equation. $gT^2 = 2R\tan\alpha$

Where α is the angle of projection

From equations (9) and (10)

T g = 2Usinα ….(a) ,

R g = 2 U^2 sinα cos α …….(b)

Eqn (a)^2 ÷ eqn (b)

$$\frac{(Tg)^2}{R\,g} = \frac{4U^2\sin^2\alpha}{2U^2\sin\alpha\ \cos\alpha}$$

$$\frac{T^2 g}{R} = \frac{2\sin\alpha}{\cos\alpha}$$

Hence $T^2 g = 2R\tan\alpha$

2. Two footballers, 120m apart, stand facing each other. One of them kicks a ball from the ground such that the ball takes off at a velocity of $30ms^{-1}$ at 38^0 to the horizontal. Find the speed at which the second footballer must run towards the first footballer in order to trap the ball as it touches the ground, if he starts running at the instant the ball is kicked.

For the first footballer, the time he ball takes to touch the ground is

c) $T = \dfrac{2u\sin\theta}{g}$

$$= \frac{2x30 \times \sin 38}{9.8}$$

$$= 3.78s$$

$$R = \frac{u^2 \sin 2\alpha}{g}$$

$$R = \frac{30^2 \times \sin 76}{9.8}$$

$$R = 89.1$$

The time taken by the second footballer to reach the ball is 3.78s.

The distance travelled by the second footballer is s = 120 – 89.1 = 30.9m

Therefore the speed of the second footballer distance / time = 30.9/3.78 = $8.2ms^{-1}$

3. A projectile is fired from ground level with a velocity of $500ms^{-1}$, 30^0 to the horizontal. Find the horizontal range, the greatest height to which it rises and time taken to reach the greatest height. What is the least speed with which it could be projected in order to achieve the same horizontal range?

$u = 500ms^{-1}$ $\qquad \alpha = 30^0$

(i) Range $= \dfrac{u^2 \sin 2\alpha}{g}$

$$= \frac{500^5}{9.81}\sin(2x30)$$

$$= \underline{\underline{22069.96m}}$$

(ii) $H = \dfrac{u^2 \sin^2 \alpha}{2g}$

$$= \frac{500^2 (\sin 30)^2}{2 \times 9.81}$$

= 3185.5m

(iii) Time taken to reach the greatest height.

$$T = \frac{u \sin \alpha}{g}$$

T = (500sin30)/9.81 = 25.5s

(b) $U_{\min} = (Rg)^{1/2}$

$(22069.96 \times 9.81)^{1/2}$

$\underline{\underline{465.3ms^{-1}}}$

Exercise 8:

(1) A body is thrown from the top of a tower 30.4m high with a velocity of 24ms $^{-1}$ at an elevation of 3 0^0 above the horizontal. Find the horizontal distance from the roof of the tower of the point where it hits the ground.

(2) A body is projected at such an angle that the horizontal range is three times the greatest height. Given that the range of projection is 400m, find the necessary velocity of projection and angle of projection.

(3) A projectile fired at an angle of 60^0 above the horizontal strikes a building 30m away at a point 15m above the point of projection.(i) Find the speed of projection.
(ii) Find the velocity of the projectile when it strikes the building.

4. An object P is projected upwards from a height of 60m above the ground with a velocity of $20ms^{-1}$ at 30^0 to the horizontal. At the same time, an object Q is projected from the ground upwards towards P at 30^0 to the horizontal. P and Q collide at a height 60m above the ground while they are both moving downwards. Find,

(i) The speed of projection of Q.

(ii) The horizontal distance between the points of projection.

(iii) The kinetic energy of P just before the collision with Q if the as of P is 0.5 kg.

NEWTON'S LAWS OF MOTION

Law 1. A body stays at rest or if moving, it continues to move with a uniform velocity unless it is acted on by an external force.

The 1st law is sometimes called the law of inertia

Inertia is the reluctance of a body to start moving if it is at rest, or to stop if it is already moving.

Inertia of the body increaser with mass. The effect of inertia can be observed by passengers in a bus. There is a forward jerk when the vehicle stops and a backward jerk when the car starts.

Linear momentum of the body is the product of its mass and its velocity

$$\therefore P = mv \qquad \text{units of P} = \text{kgms}^{-1}$$
$$[p] = [m].[v]$$
$$= MLT^{-1}$$

The rate of change of momentum is directly proportional to the resultant force and it is in the direction of the force

$$\frac{dp}{dt} \propto F.$$

F= $k\frac{dp}{dt}$, where K is a constant. But P = mv

$$\therefore \text{F} = \frac{\text{kd}}{\text{dt}}(mv)$$

If m is constant :-

$$\text{F} = \text{km}\,\frac{\text{dv}}{\text{dt}}$$

$$\text{But } \frac{\text{dv}}{\text{dt}} = a$$

$$\therefore F = kma$$

A force of 1N acting as a mass of 1kg gives the mass an acceleration of 1ms^{-2}

If F= IN , m= 1kg, a=1ms^{-2}

$\therefore 1 = k \times 1 \times 1$

$but k = 1$

$\therefore F = ma = m\dfrac{dv}{dt}$

Law 3

Action and reaction are equal and opposite e.g. when two objects interact with each other the force exerted by the 1st body on the second body is equal and opposite to the force exerted by the 2nnd body on the 1t body.

Example

1. A block of mass 2kg is pushed along a table with constant velocity by force of 5N. when the push is increased to 9N, what is the resultant force and acceleration?

$\text{Resultant force}\, F = 9 - 5 = 4N$

$\text{But } F = ma$

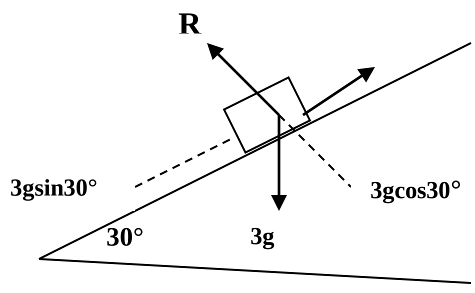
$4 = 2a$

$a = 2ms^{-2}$

2. A body of 3kg slides down a plane which is inclined at 30^0 to the horizontal. Find the acceleration of the body if

(a) The plane is smooth

(b) There is a frictional resistance of 9N.



R is the normal reaction

a)

$F = ma$

$3g \sin 30^0 = 3a.$

$a = 4.9sm^{-1}$

$F = ma$

b) $F = 3g \sin 30^0 - 9 = 3a$

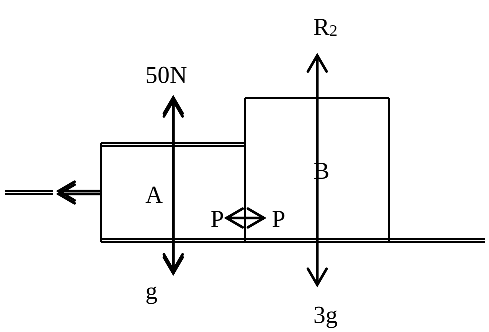
$a = 1.9ms^{-1}$

Note friction force acts in the opposite direction of motion.

3. Two blocks, A of mass 1kg and B of mass 3kg, are side by side and with contact with each other. They are pushed along the smooth flow under the action of a constant force 50N applied to A. Find

i) The acceleration of the blocks

ii) The force exerted on B by A.



$F = ma$

$50 = (1+3)a$

$a = 12.5ms^{-1}$

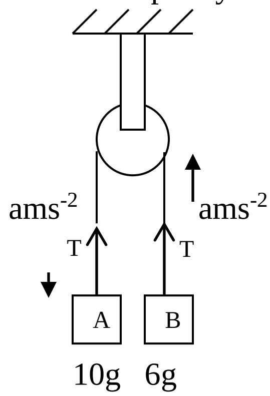
Using A

$50 - p = (1 \times 12.5)$

$p = 50 - 12.5$

$p = 37.5N$

4. A light cord connects 2 objects of masses 10kg and 6kg respectively over a light frictionless pulley. Find the acceleration and tension in the cord



Body A

$10g - T = 10a$ ………… (i)

Body B

$T - 6g = 6a$…………(ii)

Eqn(i) – eqn (ii)

$4g = 16a$

$a = (1/4)g = 2.45ms^{-2}$

Acceleration, $a = 2.45ms^{-2}$

From eqn (ii) $T=6x2.45 +6x9.81 = 73.6N$

Exercise 8

1. The car of mass 1000kg tows a caravan of mass 600kg up a road which uses 1 metre vertically for every 20 metres of its length. There constant frictional resistance of 200N and 100N to the motion of the car and caravan respectively. The combination has an acceleration of $1.2ms^{-2}$ with the engine on constant driving force.

Find

(i) The driving force.

(b) The tension in the tow bar.

2. A rectangular block of mass 10 kg is pulled from rest along a smooth inclined plane by a light inelastic string which passes over a light frictionless pulley and carries a mass of 20kg. The inclined plane makes an angle of 30^0 with the horizontal.

Determine

(i) The acceleration of the block

(ii) The tension in the string

(iii) The K.E of the block when it has moved 2m along the inclined plane.

Impulse

The product of the net force and the time interval during which the force acts is called the impulse

If a steady force F acting on a body of mass in increases the velocity of the body from u to v in the time Δt, the average acceleration

$$\vec{a} = \frac{\vec{v} - \vec{u}}{\Delta t}$$

From Newton's second law:

$$\vec{F} = m\vec{a}$$

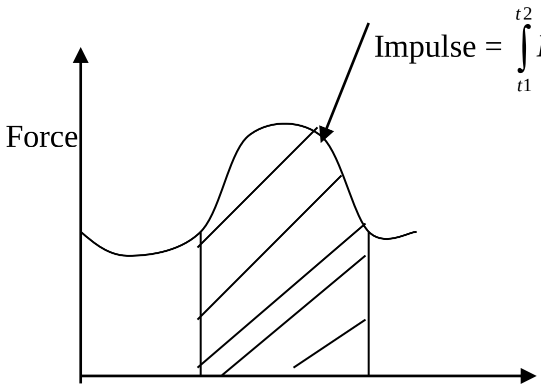
$$\vec{F} = m\left(\frac{\vec{v} - \vec{u}}{\Delta t}\right)$$

$$\vec{F}\,\Delta t = \rightarrow \rightarrow = m(\vec{v} - \vec{u})$$

In general, impulse = $\int_{t1}^{t2} F dt$

Where $\vec{v}_1$ and $\vec{v}_2$ the velocities at are times t_1 and t_2

Impulse is the area under the force time curve.



t_1 t_2 Time

Impulse is a vector quantity.

Units of impulse: NS

[Impulse] = MLT^{-2}xT

= MLT^{-1}

Example:

A tennis ball has a mass of 0.07kg. It approaches a racket with a speed of 5ms^{-1} and bounces off and returns the way it came from with a speed of 4ms^{-1}. The ball is in contact with the racket for 0.2 seconds. Calculate:

i) The impulse given to the ball.

ii) The average force exerted on the ball by the racket

i) Impulse = Ft

$$Ft = m(v - u)$$
$$= 0.07(-4 - 5)$$
$$= 0.07x - 9$$
$$= -0.63Nm$$
$$F = m\left(\frac{v - u}{t}\right)$$
$$= \frac{0.63}{0.2}$$
$$= 3.15N$$

COLLISIONS

Principle of conservation of linear momentum.

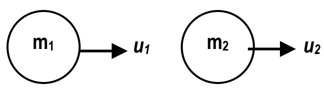
When two or more bodies collide, the total momentum of the system is conserved provided there is no external force acting on the system.

Proof

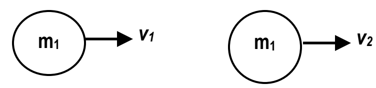
Consider a body of mass m_1 moving with a velocity u_1 to the right. Suppose the body makes a head on collision with another body of mass m_2 moving with velocity u_2 in the same direction.

Let v_1 and v_2 be the velocities of the 2 bodies respectively after collision

Before collision



After collision



Let F_1 be the force exerted on m_2 by m_1 and F_2 the force exerted on m_1 by m_2. using Newton's 2nd law.

$F_1 = M_1\left(\frac{v_1 - u_1}{t}\right), F_2 = m_2\left(\frac{v_2 u_2}{t}\right)$ t is the time of collision

Using Newton's third law

$$F_1 = -F_2$$

$$m_1\left(\frac{v_1 - u_1}{t}\right) = -m_2\left(\frac{v_2 - u_2}{t}\right)$$

$$m_1 v_1 - m_1 u_1 = -m_2 v_2 + m_2 u_2$$

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

Hence: total momentum before collision = total momentum after collision.

Types of collision

There are three types of collision

- Elastic collision

- Inelastic collision
- Perfectly inelastic collision

Elastic	**Inelastic**	**Perfectly inelastic**
Momentum conserved	Momentum conserved	Momentum conserved
Kinetic energy is conserved	Kinetic energy not conserved	Kinetic energy not conserved
		After collision the particles move together

Elastic collision

Momentum is conserved

$m_1u_1+m_2u_2 = m_1v_1+m_2v_2$

$m_1 (u_1-v_1) = m_2 (v_2-u_2)$…………………(i)

Kinetic energy is conserved

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1 + \frac{1}{2}m_2v_2^2$$

$$m_1\left(u_1^2 - v_1^2\right) = m_2\left(v_2^2 - u_2^2\right)\ldots\ldots\ldots\ \ldots\ldots\ldots\ ..(ii)$$

Equation (i) :- (ii)

$$\frac{m_1(u_1 - v_1)}{m_1\left(u_1^2 - v_1^2\right)} = \frac{m_2(v_2 - u_2)}{m_2\left(v_2^2 - u_2^2\right)}$$

$$\frac{u_1 - v_1}{(u_1 + v_1)(u_1 - v_1)} = \frac{v_2 - u_2}{(v_2 + u_2)(v_2 - u_2)}$$

$$\frac{1}{u_1 + v_1} = \frac{1}{v_2 + u_2}$$

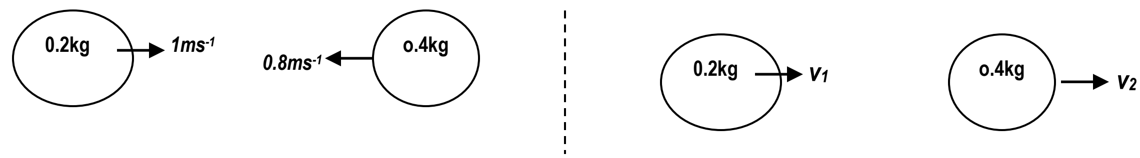
$$u_1 + v_1 = v_2 + u_2$$

$$(u_1 - u_2) = (v_2 - v_2)$$

$$\text{OR } (\mathrm{u}_2 - u_1) = (v_2 - v_1)$$

Example

1. A 200g block moves to the right at a speed of $100cms^{-1}$ and meets a 400g block moving to the left with a speed of $80cms^{-1}$. Find the final velocity of each block if the collision is elastic.



$$(v_2 - v_1) = -(-0.8 - 1)$$
$$v_2 - v_1 = 1.8...............(i)$$

using conservation of momentum.

$$m_1v_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$(0.2\times 1) + (-0.4\times 0.0) = (0.2)v_1 + (0.4)v_2$$
$$-0.12 = 0.2v_1 + 0.4v_2$$
$$-0.6 = v_1 + 2v_2....................(2)$$

$$v_2 - 1.8 + v_1$$
$$-0.6 = v_1 + 1.8 + v_1$$
$$v_1 = -1.2ms^{-1}$$
$$v_2 = 0.6ms^{-1}$$

2. A neutron of mass m makes a head on elastic collision with a stationary atomic nucleus of mass 12m with a velocity u.

Calculate:

i. the fractional decrease in the kinetic energy of the neutron

ii. The velocity of the nucleus after the collision

$m_1u_1+m_2u_2 = m_1v_1+m_2v_2$

$u_{2 = 0}$

$m_1u_1 \quad = m_1v_1+12m_2v_2$

$m_1 = m$, $m_2 = 12m$

$u_1 = v_1+12v_2$................(i)

From conservation of kinetic energy

$v_2 - v_1 = u_1 - u_2$

$v_2 - v_1 = u_1$.....................(ii)

From (i) and (ii)

$v_2 - v_1 = v_1 + 12v_2$

$v_2 - 12v_2 = v_1 + v_1$

$-11v_2 = 2v_1$

$v_2 = \frac{2}{-11} v_1 ....(iii)$

put (iii) in (ii)

$$\frac{-2}{11} v_1 \frac{v_1}{1} = u_1$$

$$\frac{-2v_1 - 11v_1}{11} = u_1$$

$$\frac{-13v_1}{11} = u_1$$

$$-13v_1 = 11u.$$

$$v_1 = \frac{11u}{-13}$$

$$v_1 = -0.85u.$$

fractional decrease $= \dfrac{\frac{mu^2}{2} - \text{Kineticenergy after collision}}{\frac{\text{mu}^2}{2}}$

Kinetic energy after collision

$$= \frac{1}{2} mv^2$$

$$= \frac{1}{2}.m.\left(\frac{11v}{13}\right)^2$$

$$= \frac{m}{2} . \frac{121v^2}{169}$$

$$\frac{mv^2 121}{338}$$

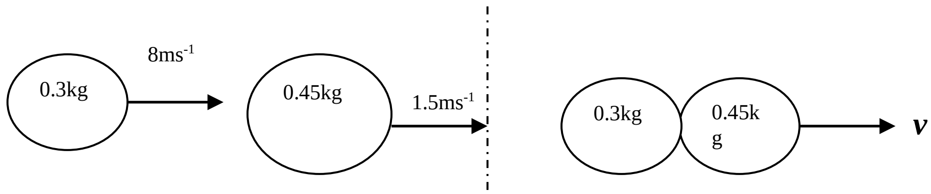
$$\therefore \frac{\frac{mu^2}{2} - \frac{mv^2 121}{338}}{\frac{mu^2}{2}}$$

$$= \frac{169 - 121}{169}$$

$$= 0.28$$

$$= \frac{7}{25}$$

3. A bullet of mass 300g travelling horizontally at a speed of $8ms^{-1}$ hits a body of mass 450g moving in the same direction as the bullet at $1.5ms^{-1}$. The bullet and the body move together after collision. Find the loss in kinetic energy.



$$m_1v_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(0.3 \times 8) + (1.5 \times 0.45) = (0.3 + 0.45)v$$

$$v = \frac{3.075}{0.75}$$

$$v = 4.1ms^{-1}$$

loss in kinetic energy : $\left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1 + m_2)v^2$

$$\left(\frac{1}{2} \times 0.3 \times 64 + \frac{1}{2} \times 0.45 \times 2.25\right) - \frac{1}{2}(0.75)16.81$$

$$10.12 - 6.30$$

$$3.82\,joules$$

Exercise

1. An object *A* of mass *m* moving with a velocity of $10ms^{-1}$ collides with a stationery object *B* at equal mass *m*. After collision A moves with a velocity U at an angle of 30^0 to its initial direction and B moves with a velocity V at an angle of 90^0 to the direction U.

   i) Calculate the velocities U and V (U = $5\sqrt{3}$ ms^{-1}, V = $5ms^{-1}$)

   ii) Determine whether the collision was elastic or not.(Kinetic energy before collision = kinetic energy after collision = 50m, hence collision is elastic)

2. A body of mass 5.0kg is moving with a velocity 2.0ms^{-1} to the right. It collides with a body of mass 3.0 kg moving with a velocity of 2.0ms^{-1} to the left. If the collision is head-on and elastic, determine the velocities of the two bodies after collision.(-1.0 ms^{-1}, 3.0ms^{-1})

3. A car of mass 1000kg travelling at uniform velocity of 20ms-1, collides perfectly inelastically with a stationary car mass 1500kg. Calculate the loss in kinetic energy of the cars as a result of the collision.(1.2×10^5J)

SOLID FRICTION

There are 2 types of friction i.e.

(i) Static friction

(ii) Kinetic friction / sliding friction

Static friction opposes the tendency of one body sliding over the other. Kinetic opposes the sliding of one body over the other.

Limiting friction is the maximum friction between on two surfaces.

Laws of solid friction

(i) Frictional force between 2 surfaces always oppose their relative motion or attempted motion.

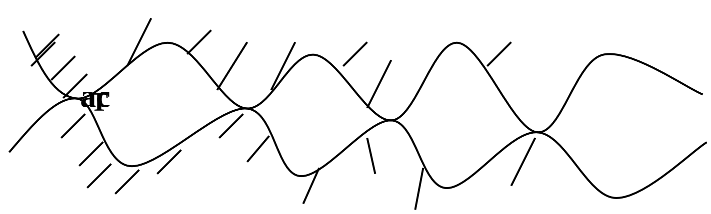
(ii) For given pair of surface in contact, the limiting frictional force is proportional to the normal reaction.

(iii) For two surfaces in contact, the sliding frictional force is proportional to the normal reaction and independent of the relative velocity of these surfaces.

(iv) The frictional force is independent of the area of contact of the given surface provided the normal reaction is constant.

Molecular Theory and the laws of solid friction.

On a microscopic level, even a highly polished surface has bumps and hollow. It follows that when 2 surfaces are put together, the actual area of contact is less than the apparent area of contact



At points of contact like a,b,c, small cold-welded joints are formed by the strong adhesive forces between the molecules in the two surfaces.

These joints have to be broken before one surface can move over the other.

This accounts for law 1.

The actual area of contact is proportional with the normal force (reaction). The frictional force which is determined by the actual area of contact at the joints is expected to be proportional to the normal force.

This accounts for law 1 and 3

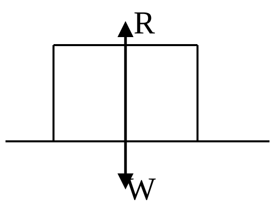
If the apparent area of contact of the body is decreased by turning the body so that it rests on one of the smaller side, the number of contact points is reduced. Since the weight of the body has not altered, there is increased pressure at the contact points and this flattens the bumps so that total contact area and the pressure return to their original values.

Therefore, although the apparent area of contact has been changed, the actual area of contact has not.

This accounts for law 4

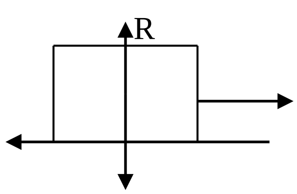
Coefficient of static friction

Consider a block resting on a horizontal surface



The block is in equilibrium under the action of its weight W and normal reaction R.

Suppose a string is fixed to the block and the tension (T) in the string increased gradually, the static frictional force Fs; which posses the tendency of the block to ride over the surface comes in play. In equilibrium Fs=T.



Fs

w

The value F_l of Fs at which the block starts moving is called the limiting frictional force $(0 < F_s \leq F_l)$

The ration of the limiting frictional force to the normal reaction is called the co-efficient of static friction μ_s

$$\mu_s = \frac{F_l}{R}$$

$$F_s \leq F_l = \mu_s R$$

$$F_s \leq \mu_s R$$

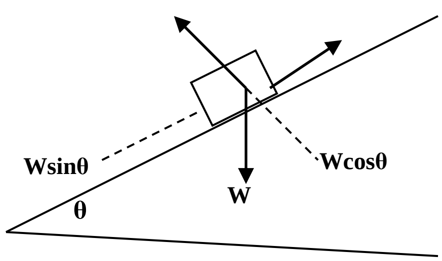
$$0 \leq \mu_s \leq 1$$

Measurement of coefficient of static friction, μ_s

Method 1: Using a tilting plane.

A block is placed on a plane and the plane is tilted until when the block begins to slide. The angle of θ of inclination of the plane surface to the horizontal is measured.

The co-efficient of friction is given by $\mu_s = \tan\theta$



When the block is at the point of sliding

Fs = Wsinθ ………….(i)

R = Wcosθ…………..(ii)

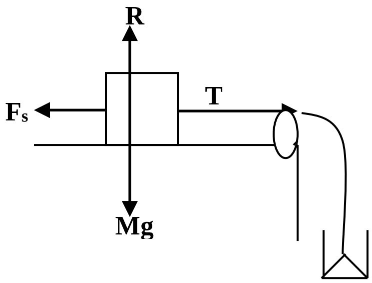
$(i) \div (ii)$

$$\frac{Fr}{R} = \frac{W\sin\theta}{W\cos\theta}$$

$$but\frac{Fr}{R} = \mu_s$$

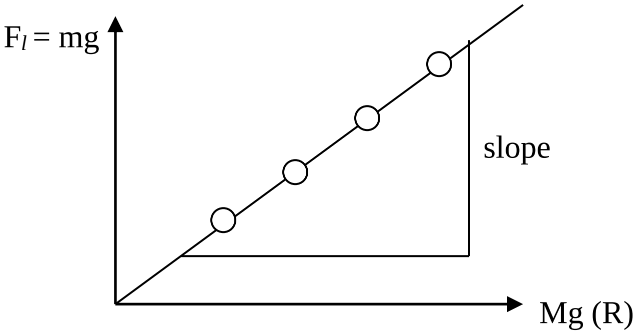
$$\mu s = \tan\theta$$

Method 2: To determine the co-effienct of static friction.



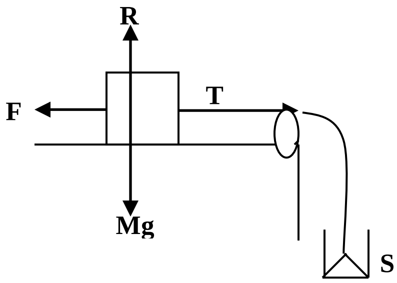
Masses are added to the scale pan until the block just slides. The total mass m of the scale pan and masses added is noted. The prodecures is repeated for different values of R obtained by adding known weights to the block.

A graph of mg against R(Mg) is plotted.



The slope of the graph is μ_s

Co-efficient of kinetic (dynamic friction.



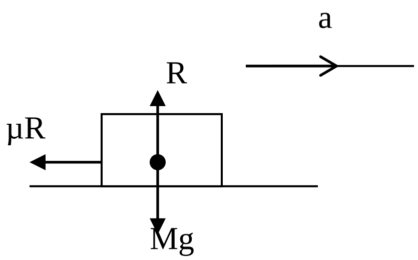
Weights are added to a scale pan S and each time, body is given a slighht push.

At one stage, body continues to move with a constant velocity and kinetic frictional force F is then equal to the weight of the scale pam together with the pan’s weight.

On dividing F by the weight of body, the co-efficient of dynamic friction canbe calculated.

Example 1

A car of mass 200kg moving along a straight road at a speed of 96kmh^{-1} is brough to rest by steady application of the brakes in a distance of 80m. find the co-efiecient of kinetic friction between the tires and the road.



$$ma = -\mu R$$
$$ma = -\mu mg$$
$$a = -\mu g$$
$$\mu = \frac{-a}{g}$$

$$u = 96kmh^{-1} \rightarrow 96 \times \frac{5}{18} ms^{-1} = 26.7ms^{-1}$$
$$v = 0$$
$$s = 80m$$
$$a = ?$$

$v^2 = u^2 + 2as$

$0 = (26.7)^2 + (2 \times a \times 80)$

$a = -\frac{26.7^2}{160}$

$a = -4.5ms^{-2}$

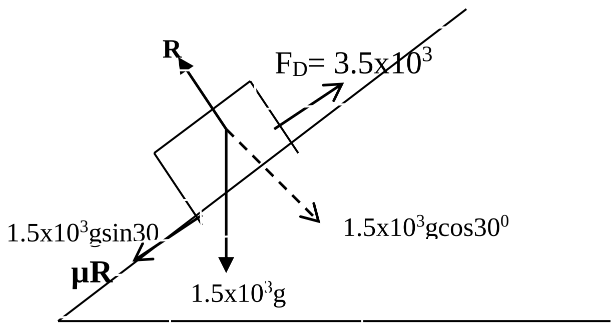
$\therefore \mu = \frac{4.45}{9.8}$

$= 0.45$

A car of mass 1.5 x 10^3kg and tractive pull 3.5x10^3N climbs a truck which is inclined at an angle of 30^0 to the horizontaa. The speed of the car at the bottom of the incline is 20ms^{-1} and the coffient of sliding friction is 0.25, calculate

(i) The distance travelled along the incline before the car comes to a halt.

(ii) The time taken ttravelling along the incline before the car comes to a halt.



$F = ma$

$(3.5 \times 10^3) - (1.5 \times 10^3 g \sin 30 + 0.25 \times 1.5 \times 10^3 g \cos 30^0) = 1.5 \times 10^3 a$

$(3.5 \times 10^3) - (750 + 324.8) = 1.5 \times 10^3 a$

$a = -4.69ms^{-2}$

$u = 20ms^{-1}$

$a = -4.69ms^{-2}$

$v = 0$

$s = ?$

$V^2 = u^2 + 2as$

$0 = (20^2) + (2 \times -4.69 \times s).$

$400 = 9.38s.$

$s = 42.6m$

ii) $S = 42.6$
$a = -4.96$
$u = 20$
$v = 0$
$t = ?$

$v = u + at$
$0 = 20 - 4.96t$
$20 = 4.96t.$
$t = 4s$

An old car of mass 1500kg and tractive pull 4000N climbs a tract which is incllined at an angle of 30^0 to the horizontal. The velocity of the car at the bottom of the incline is $108kmh^{-1}$ and the co-efiecient of sliding friction is 0.35.

(i) Clculate the distance travelled along the incline before the car comes to a halt.(86.53m)

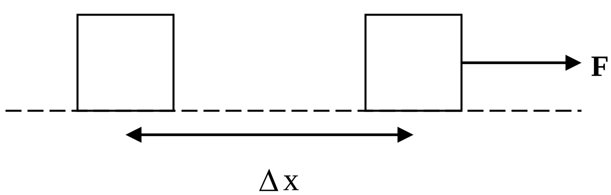
(ii) The time taken to travel along the incline before the car comes to a halt.(5.77s)

WORK, POWER AND ENERGY

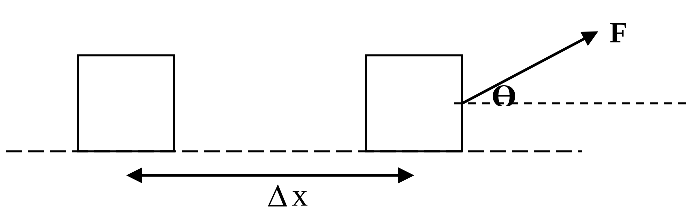
work done by a constant force

Work is defined as the product of the distance moved by the point of aplication of the force and the component of the force in the direction of motion.

Consider a body by mass in resting on a smooth surface.



If a force F moves the object through a distance X, then work done w = F.Δx If the force puls the block at angle Θ to the horizontal through a horizontal distance Δx



Work done, w = (FcosΘ) Δx

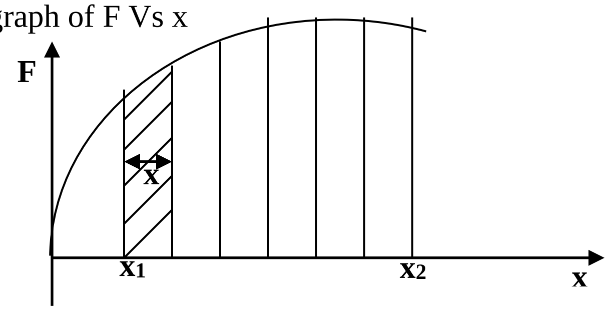
Work is a scalar quantity.

S.I unit J.

<u>Work done by a variable force</u>

Consider a force F = f(x) which val=ries in magnitude

A graph of F Vs x



If it is required to find the work done by the force when its point of application moves from x = x_1 and x = x_2 then the interval $x_1 \rightarrow x_2$ is subdivided into small displacements, $\Delta x_{1,} \quad \Delta x_2 .... \Delta x_n$

The work done by the force during the displacement

$\Delta x_1 is F_1 \Delta x_2$ (since Δx_1 is too small, the F_1 can be considered constant)

For another short interval

Δx_2 work done $= F_2 \Delta x_2$. therefore, work done during the displacement

$x_1 \rightarrow x_2 \Delta W = F_1 \Delta x_1 + F_2 \Delta x_2 + F_3 \Delta x_3 + ... F_n \Delta x_n = \sum_{i=1}^{n} F_1 \Delta x_i$

For $n \rightarrow \infty$

$$W_R = \frac{\lim}{\Delta xi \rightarrow 0} \sum_{i \rightarrow 1}^{n} Fi \Delta xi$$

$$= \int_{x_1}^{x_2} F.dx$$ = area under the force -distance grgaph

in vector form, work done by a variable force is given by

$$w = \int_{r_1}^{r_2} \vec{f} . \vec{dr}$$

where r is the displacement.

Work – energy theorem

Variable force

Consider an external force F= F(x) which acts as a mass m giving it an acceleration a, by Newton's second law F = ma $= {mdv}/{at}$. The work , ∂x ,done in displacing mass m, throguh a small distance ∂x , under action of a force F.

$$\partial w = F\partial x$$

total work done = $\int_{x_1}^{x_2} Fdx.$

$$\int_{x_1}^{x_2} m\frac{dv}{dt}dx$$

$$\text{but } \frac{\text{dx}}{\text{dt}} = \text{v}$$

$$\text{w} = \int_{\text{v}_1}^{\text{v}_2} \text{mv dv}$$

where v_1 and v_2 are the velocities of the body when at dispacement x_1 and x_2 respectively.

$$W = \left[\frac{mv^2}{2}\right]_{v_1}^{v_2}$$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2mv_1^2}$$

The above is the expression for the work –energy theorem

It states that, *the work done by the resultant external force is equal to the change in the kinetic energy of the body.*

Constant force

Consider a mass in initially moving of a speed *u* which is subjected to a constant retarding force F. suppose the speed is reduced to v in a distance S

Using
$$v^2 = u^2 + 2as$$
$$as = \frac{v^2 - u^2}{2}$$

Work done by the retarding force

= -FS

= -mas

$$\text{but as} = \frac{\text{v}^2 - u^2}{2}$$
$$W = -m\left(\frac{v^2 - v^2}{2}\right)$$
$$W = \frac{mu^2}{2} - \frac{mv^2}{2}$$

Again work done = change in kinetic energy.

Gravitational potential energy

Suppose a body of mass m is raised from a height y_1 to a height y_2 above the surface of the earth, the work done by the gravitational force when the body is raised through a small height $\partial y; \partial w = F\partial y$

Where F= gravitational force = -mg

$$\partial w = -mg\partial y$$

Work done to raise the body from height y_1 to height y_2 is

$$\int_{y_1}^{y_2} -mg \quad \text{dy}$$
$$= -mg\int_{y_1}^{y_2} dy$$
$$= mg\left[y\right]_{y_1}^{y_2}$$
$$\therefore W = -(mgy_2 - mgy_1).$$

from work -energy theorem

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$w = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -mgy_2 + mgy_1$$

$$\frac{1}{2}mv_2^2 + mgy_2 = mgy_1 + \frac{1}{2}mv_1^2$$

The term mgy = gravitational potential energy.

$$\frac{1}{2}mv^2 + mgy = constant .... *$$

Hence $Potential energy + Kinetic energy = \text{mechanical energy}$

$\text{Equation} * \text{implies that merchanical energy is conserved.}$

Principal of conservation of mechanical energy

In a given system where the only force acting are conservative forces, the sum of Kinetic energy and Potential energy is constant.

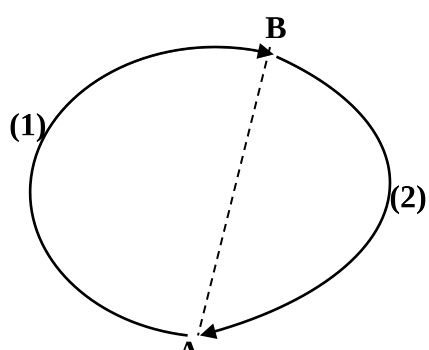
Question

Show that the following obey the law of mechanic energy.

(i) A swinging pendulum

(ii) A falling stone.

Conservative forces.



$W_{AB}^{(1)} = W_{AB}^{(2)}$, then the force being used is a conservative force.

For a conservative force, the work done is independent on the path taken.

Work done when a body moves round a closed path is zero i.e. $W_{AB}^{(1)} + W_{AB}^{(2)} = 0$

Let $W_{AB}^{(1)}$ be the work done to move the mass from A to B via path 1 and $W_{AB}^{(2)}$ be the work done to move the mass from A to B via path (2).

If $W_{AB}^{(1)} = W_{AB}^{(2)}$ then the work done is independent of the path taken in the field of force.

Examples of conservative forces; Gravitational force, Elastic force, Electrostatic force.

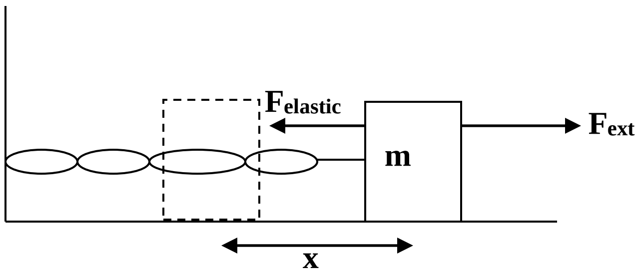
For a conservation force, the work done in moving the body round a close path in the field of force is zero.

In a conservative force field, mechanical energy is conserved

Non- conservative forces: In a conservative force, the work done by a non conservative force round a closed path is not zero and is dependent on the path taken.Example of non-conservative forces: Friction, Air resistance, Viscosity drag.

<u>Elastic potential energy</u>

Consider a mass m resting on a smooth horizontal surface and attached to a spring whose other end is fixed.



Suppose an external force F_{ext} is applied to the mass, so that the spring becomes stretched by a distance x. An equal and opposite force, $F_{elastic}$ i.e elastic force appears in the spring.

$F_{elastic}$ = kx (Hooke's law)

Force is directly proportional to extension provide the elastic limit is not exceeded.

k= force constant , F_{ext} = kx

When a spring is stretched from $x = x_1$ to $x = x_2$,

$$W = \int_{x_1}^{x_2} F_{elastic} . dx$$

$$-\int_{x_1}^{x_2} kx \; dx$$

$$w = -\left[\frac{kx_2^2}{2} - \frac{kx_1^2}{2}\right]$$

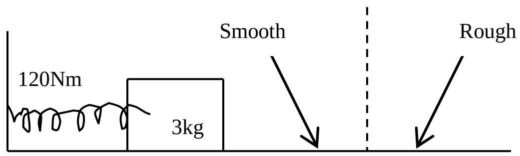
from work – energy theorem

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -\left(\frac{kx_2^2}{2} - \frac{kx_1^2}{2}\right)$$

$$\frac{1}{2}mv_2^2 + \frac{kx_2^2}{2} = \frac{1}{2}mv_1^2 + \frac{kx_1^2}{2}$$

then term V(x) = $\frac{1}{2}kx^2$ is the elastic potential energy.

A 3.0kg block is held in contact with a compressed spring of a force constant $120Nm^{-1}$.The block rests on the smooth portion of a horizontal surface which is partly smooth and partly rough as shown.



When the block is released, it slides without friction until it leaves the spring and then continues to move along the rough portion for 8.0m before it comes to rest. The coefficient of sliding friction between the block and the rough surface is 0.20. Calculate the: (i) maximum kinetic energy the block.

(iii) Compression of the spring before the block was released.

Solution

Kinetic energy = work done against frictional force

$$\frac{1}{2}mv^2 = \mu \text{mg} \times \text{distance}$$

$$\frac{1}{2}v^2 = \mu g \times 8$$

$$v^2 = (2 \times 0.20 \times 9.8 \times 8)$$

$$v = 5.6ms^{-1}$$

Kinetic energy = ½mv^2 = ½x3x $(5.6)^2$ = 47.04J

(ii) elastic energy = ½kx^2

But Kinetic energy = elastic energy

$$47.07 = \frac{1}{2} \times 120 \times x^2$$

$$x^2 = \frac{47.07 \times 2}{120}$$

$$x = 0.89m$$

2. A bullet of mass 10g is fired at short range into a block of wood of mass 990g resting on a smooth horizontal surface and attached to a spring of force constant $100Nm^{-1}$. The bullet remains embedded in the block while the spring is compressed by a distance of

5.0cm. Find the elastic energy of the compressed spring, and the speed of the bullet just before collision with the block.

Elastic energy = ½kx^2 = ½ x 100 x (0.05)^2 = 0.125J

Kinetic energy = elastic energy

$$\frac{1}{2}mv^2 = 0.125$$
$$(0.01 + 0.99)v^2 = 0.25$$
$$v = 0.5ms^{-1}$$

Using conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$(0.01 \times u) + (0.99 \times 0) = 0.5(0.01 + 0.99)$$
$$u = 50ms^{-1}$$

Exercise

1. A mass of 500g is released from rest so that if falls vertically through a distance of 20cm onto a scale pan, of negligible mass, hung from a spring of force constant 100Nm^{-1}. Find the position of the scale pan when it first comes to rest. (0.14m)

POWER

It is the rate of doing work.

$$power = \frac{dw}{dt} = \frac{d}{dt}(F.S)$$ where w is work done, F is force , S is distance travelled.

For constant force

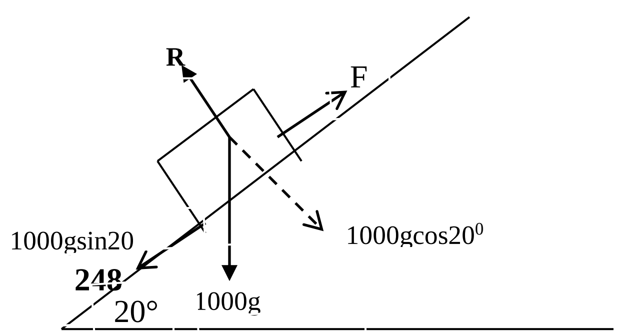
$p = F.\frac{ds}{dt}$ But $\frac{\text{ds}}{\text{dt}} = v$

$\therefore p = FV$

Unit of power is watts

Example

1. A particle of mass 1000kg moves with uniform velocity of $10ms^{-1}$ up a straight truck inclined at an angle of 20° to the horizontal. The total frictional resistance to motion of the car is 248N. Calculate the power developed in the engine.



$F = 1000g\sin 20 + 248$

$3599.8N$

$P = Fv$

(3599.8×10)

$= 35997.9W$

$= 36kw$

2. Sand is deposited at a uniform rate of $20kgs^{-1}$ and of negligible kinetic energy onto an empty conveyor belt moving horizontally at a constant speed of 10m / minute.

Find

(i) A force required to maintain a constant velocity.

(ii) The power required to maintain a constant velocity

(iii) The rate of change of K.E of the moving sand

(iv) Why are the latter 2 quantities unequal?

$$v = \frac{10}{60} = \frac{1}{6}\,\mathrm{ms}^{-1}$$

$$\mathrm{F} = \frac{\mathrm{dp}}{\mathrm{dt}} \;,\; \mathrm{p} = \mathrm{mv}$$

$$\mathrm{F} = \frac{\mathrm{d(mu)}}{\mathrm{dt}} \qquad v = \frac{dm}{dt}$$

$$\frac{dm}{dt} = 20kgs^{-1}$$

$$F = \frac{1}{6} \times 20$$

$$= 3.33N$$

(ii)

$$P = FV$$

$$= \frac{20}{6} \times \frac{1}{6}$$

$$= \frac{20}{36}\,\mathrm{W}$$

$$\mathrm{W} = 0.56\mathrm{w}$$

Rate of Kinetic Energy

$$= \frac{1}{2}\left(\frac{dm}{dt}\right)v^2$$

$$= \frac{1}{2} \times 20 \times \left(\frac{1}{6}\right)^2$$

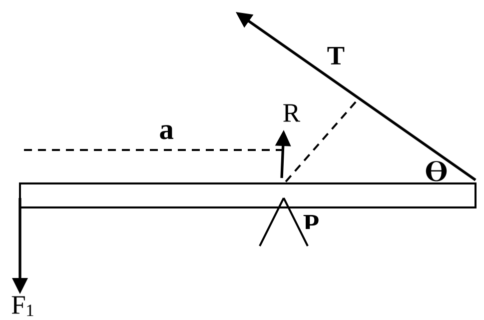
$$= 0.28J$$

The two quantities are not equal because there is a frictional force that has to be overcome.

MOMENTS AND COUPLES

Moment of a force

The moment of a force about an axis is the product of the force and the perpendicular distance from the axis to the line of action of the force.



Moment of F_1 about P

$= F_1a$

Moment of R about P = 0

Moment of T about P = T. a sin Ө

Principal of moments

If a body is in equilibrium, under the action of a number of force, the algebraic sum of the moment of the forces about any axis is zero i.e. total clockwise moments = total anticlockwise moments about the same axis.

Conditions for equilibrium

(i) Translational equilibrium.

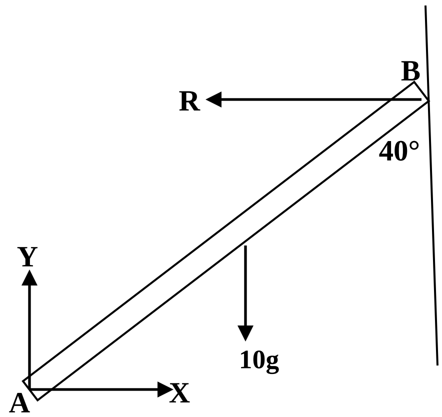
The resultant force must be zero i.e. sum of forces in one direction should be equal to sum of forces in the opposite direction.

(ii) Rotational equilibrium

The algebraic sum of moments about any axis must be zero.

Example

A uniform rod of mass 10kg is smoothly hinged at A and rests in a vertical plane on the end B against a smooth vertical wall. If the rod makes an angle of 40^0 with the wall, find the thrust of the wall and the direction of the reaction at A



Let X and Y represent the components of the reaction in the horizontal and vertical directions respectively.

Resolving forces in the horizontal direction

R = X

Resolving forces in the vertical direction

Y = 10g = 98N

Taking moments about A:

10gx(ABsin40)/2 = R xABcos40

Therefore R = 41.1N

Hence X = 41.1N

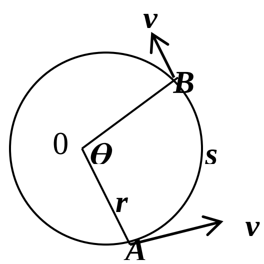
Reaction at A = $\sqrt{(41.1^2 + 98^2)}$ = 106.3N

Direction Ө = $\tan^{-1}$(98/41.1) = 67.24°

$$\text{Direction} = \text{tcm}^{-1}\left(98/_{41.1}\right)$$
$$= 67.24^0$$

CURCULAR MOTION

Consider a body moving in a circle of radius *r* with uniform speed *v*



Ssuppose the body moves from point A to point B in time't' through an angle Θ.

The angle Θ is called the *angular displacement.*

Arc length, $s = r\Theta$

Angular velocity, ω, is the rate of change of angular displacement.

$\omega = \Theta/t$

Speed,

$$v = \frac{s}{t} = \frac{r\theta}{t}, \quad \text{but } \frac{\theta}{\mathrm{t}} = \omega$$

$$v = \omega r$$

period (T), time taken to go through one circle.

When $\Theta = 2\pi$, $t = T$

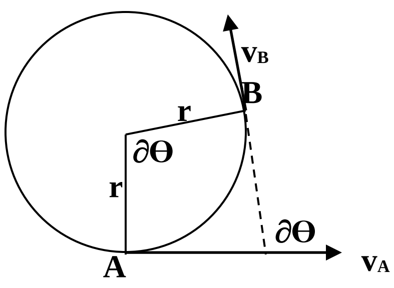
$\omega = \Theta/t = 2\pi/T$

$v = \omega r$

Therefore $v = \dfrac{2\pi}{T} r$

Acceleration of a body moving in a circle

Consider a body moving with constant speed v in a circle of radium r



If it travels from A to B in a short time, ∂t,

∂t, then arc AB $= v\partial t$

$$v = \frac{AB}{\partial t}$$

also arc AB = $r\partial\Theta$

hence

$$r\partial\sigma = v\partial t.$$

$$\partial\theta = \frac{v\partial t}{r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

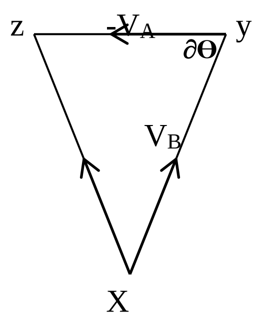
Change of velocity between A and B

$$V_B - V_A$$

$$= V_B + (^-V_A)$$

$$xZ = V_B + \left(^-V_A\right)$$

But $\left|V_A\right| = \left|V_B\right| = V$



Arc XZ = $v\partial\Theta$

From equation (1) $\partial\Theta = v\dfrac{\partial t}{r}$

Hence arc XZ = v. $v\dfrac{\partial t}{r} = v^2\dfrac{\partial t}{r}$

The magnitude of the acceleration, a, between A and B is

$$a = \frac{\text{change in velocity}}{\text{time interval}} = \frac{xz}{\partial t}$$

$$a = \frac{v^2\partial t}{r\partial t} = \frac{v^2}{r}$$

But v = ωr

a = $\omega^2 r$

The acceleration of the body moving in a circle is towards the centre of the circle.

The force on a body moving in a circle towards the centre of the circular path is called the **centripetal force**

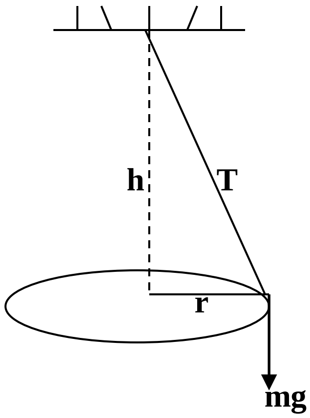
Centripetal force = ma = $m\dfrac{v^2}{r}$

Or Centripetal force = $m\omega^2 r$

Example of circular motion

Conical pendulum

Consider a body of mass *m* attached to a string of weight *l*, describing horizontal circle of radius *r* at a uniform speed *v*



$$(\uparrow) T\cos\theta = mg \text{........}(i)$$

$$(\rightarrow) T\sin\theta = \frac{mv^2}{r} \text{...................}(2)$$

$$(2) \div (1)$$

$$\tan\theta = \frac{v^2}{rg}$$

$$r = l\sin\theta$$

$$V^2 = rg\tan\theta$$

$$\text{but v} = \frac{2\pi r}{T}, T = \text{period}$$

$$\frac{4\pi^2 r^2}{T^2} = \text{rg}\tan\theta$$

$$T^2 = \frac{4\pi^2 r}{g\tan\theta}$$

$$T^2 = \frac{4\pi^2 l\sin\theta}{g\tan\theta} = \frac{4\pi^2 l\cos\theta}{g}$$

but $l\cos\Theta = h$

$$T^2 = \frac{4\pi^2 h}{g}$$

Example

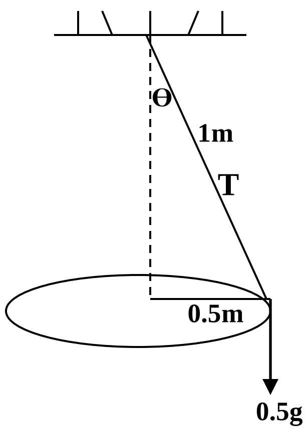
A steel ball of 0.5kg is suspended from a light inelastic string of length 1m. The ball describes a horizontal circle of radius 0.5m

Find

(i) The centripetal speed of the ball

(ii) The angular speed of the ball

(iii) The angle between the string and the radius of the circle if the angular speed is increased to such a values that the tension in the string is 10N



$$\sin\theta = \frac{0.5}{1} = 30°$$

$$(\rightarrow)\frac{mv^2}{r} = T\sin 30..................(i)$$

$$(\uparrow) T\cos 30^0 = 0.5g$$

$$T = 5.67N$$

From (i)

$$\therefore \frac{0.5v^2}{0.5} = 5.67\sin 30^0$$

$$\therefore \text{certripetal force} = \frac{\text{mv}^2}{\text{r}} = \frac{0.5\times 1.68^2}{0.5}$$

$$= 2.83N$$

(iii) Angular speed ω

$$v = wr$$
$$w = \frac{1.68}{0.5}$$
$$= 3.36 rads^{-1}$$

(iii)

$$(\uparrow) T\cos\theta = 0.5g$$
$$\cos\theta = 0.5g/10$$
$$\theta = 60.66^0$$

Exercise

1. An object of mass 0.5kg on the end of the string is whirled around in a horizontal circle of radius 2m, with a constant speed of $10ms^{-1}$. Find its angular velocity and the tension in the string.($\omega = 5rads^{-1}$, T = 25.5N)

2. A small ball of mass 0.1 kg is suspended by an inextensible string of length 0.5m and is caused to rotate in a horizontal circle of radius 0.4m. Find

(i) The resultant of these forces. (1.3N)

(ii) The period of rotation. (1.1s)

3. A pendulum bob of mass 0.2kg is attached to one end of an inelastic string of length 1.2m. The bob moves in a horizontal circle with the string inclined at 30° to the vertical. Calculate: (i) the tension in the string

(ii) the period of the motion

4. The period of oscillation of a conical pendulum is 2.0s. If the string makes an angle of 60° to the vertical at the point of suspension, calculate the:

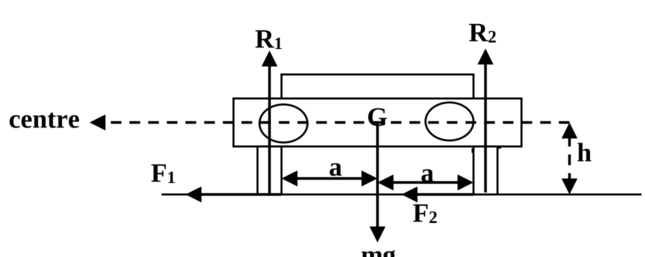
(i) Vertical height of the point of suspension above the circle.(h = 0.994m)

(ii) Length of the string, (*l* = 1.99m)

(iii) Velocity of the mass attached to the string.($v = 5.41ms^{-1}$)

Vehicle on a curved track

(i) Overturning / upsetting / toppling



consider a vehicle with mass *m* moving with a speed *v* in a circle of radius *r*; let *h* be the height of the centre of gravity above the truck and *2a* the distance between the tyres.

Resolving vertically :

$$R_1 + R_2 = mg..................(1)$$

horizontally

$$(F_1 + F_2) = \frac{mv^2}{r}.....................(2)$$

Taking moments about G:

$$R_1 a + F_1 \times h + F_2 \times h = R_2 .a$$

$$(F_1 + F_2)\frac{h}{a} = R_2 - R_1...............................(3)$$

Substitute equation (2) in equation (3)

$$\frac{mv^2}{r}.\frac{h}{a} = R_2 - R_1...............................(4)$$

Add equation (1)+ equation(4)

$$R_1 + R_2 = mg$$

$$R_2 - R_1 = \frac{mv^2 h}{ra}$$

$$\therefore \frac{2R_2}{} = \frac{m}{}\left(g + \frac{v^2 h}{ra}\right)$$

$$R_2 = \frac{m}{2}\left(g + \frac{v^2 h}{ra}\right)$$

R_2>0 implying that the outer tire never lose contact.

Equation (1)– equation(4)

$$2R_1 = mg - \frac{mv^2h}{ra}$$

$$R_1 = \frac{m}{2}\left(g - \frac{v^2h}{ra}\right)$$

When $R_1 = 0$, inner tire loses contact with the track.

$$\Rightarrow \frac{m}{2}\left(g - \frac{v^2h}{ra}\right) = 0$$

$$g - \frac{v^2h}{ra} = 0$$

$$v^2 = \frac{rag}{h}$$

$$v = \sqrt{\frac{rag}{h}}$$

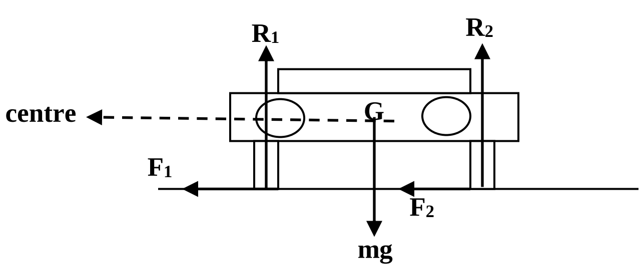
For speeds higher than $\sqrt{\frac{vag}{h}}$, the car overturns.

The vehicle is likely to overturn if

- The bend is sharp (r is small)
- The centre of gravity is high (h is large)
- The distance between the tires is small (a is small)

Skidding

A vehicle will skid when the available centripetal force is not enough to balance the centrifugal force (force away from the centre of the circle), the vehicle fails to negotiate the curve and goes off truck outwards.



For no skidding, the centripetal force must be greater or equal to the centrifugal force i.e.

$$F_1 + F_2 \geq \frac{mv^2}{r}$$

But $F_1 = \mu R_1$ and $F_2 = \mu R_2$

$$\mu(R_1 + R_2) \geq mv^2/r$$

$$\mu mg \geq \frac{mv^2}{r}$$

$$\mu g \geq \frac{v^2}{r}$$

$$v^2 \geq \mu gr$$

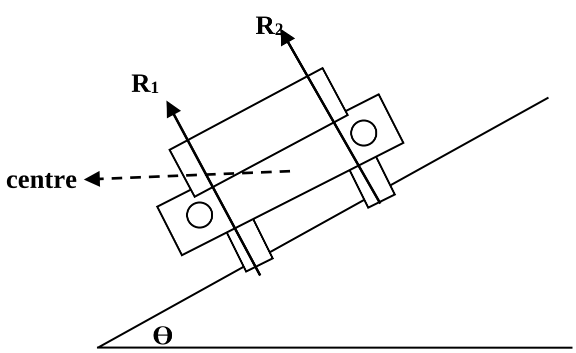
Maximum safe speed, $v_m = \sqrt{\mu rg}$

Skidding will occur if

- The vehicle is moving too fast
- The bend is too sharp (r is small)
- The road is slippery (μ is small)

BANKING OF A TRACK

- This is the building of the track round a corner with the outer edge raised above the inner one. This is done in order to increase the maximum safe speed for no skidding.
- When a road is banked, some extra centripetal force is provided by the horizontal component of the normal reaction
- When determining the angle of banking during the construction of the road, friction is ignored.



Resolving vertically

$$R_1\sin(90-\Theta) + R_2\sin(90-\Theta) = \frac{mv^2}{r}$$

But $\sin(90-\Theta) = \cos\Theta$

$(R_1+R_2)\cos\Theta = mg$ …………..(i)

Horizontally

$$R_1\cos(90-\theta) + R_2\cos(90-\theta) = \frac{mv^2}{r}$$

$$(R_1 + R_2)\sin\theta = \frac{mv^2}{r}\text{.........................}(2)$$

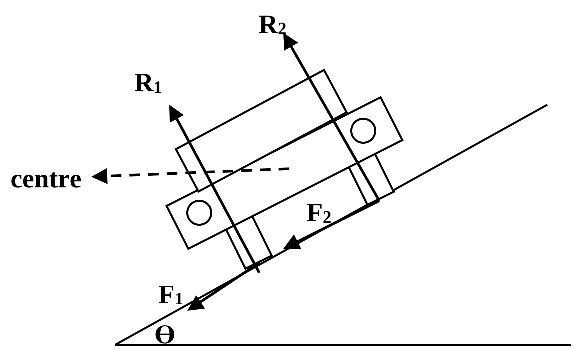
$eqn\,2 \div \text{eqn}\,1$

$$\tan\theta = \frac{\text{v}^2}{\text{rg}}$$

Hence Θ is the angle of banking

When there is friction

Suppose there is friction between the track and the vehicle moving round the bend.



Resolving vertically:

$(R_1+R_2)\cos\Theta = (F_1+F_2)\sin\Theta + mg$

$(R_1+R_2)\cos\Theta - (F_1+F_2)\sin\Theta = mg$

but $F_1 = \mu R_1$, $F_2 = \mu R_2$.

$(R_1+R_2)\cos\Theta - \mu(R_1+R_2)\sin\Theta = mg$

$(R_1+R_2)(\sin\Theta - \mu\sin\Theta) = mg$ ……………………..(1)

Horizontally

$$(\mathrm{R1}+\mathrm{R2})\sin\theta + (\mathrm{F1}+\mathrm{F2})\cos\theta = \frac{\mathrm{mv}^2}{\mathrm{r}}$$

$$(\mathrm{R1}+\mathrm{R2})\sin\theta + \mu(\mathrm{R1}+\mathrm{R2})\cos\theta = \frac{\mathrm{mv}^2}{r}$$

$$(\mathrm{R}_1+R_2)(\sin\theta+\mu\cos\theta) = \frac{mv^2}{r} \text{...............}(2)$$

$$egn\ 2 \div \mathrm{eqn}\,1$$

$$\frac{\sin\theta+\mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg}$$

$$\frac{\tan\theta+\mu}{1-\mu\tan\theta} = \frac{v^2}{rg}$$

$$v^2 = rg\left(\frac{\mu+\tan\theta}{1-\mu\tan\theta}\right)$$

$$\therefore \text{maximum safe speed} = \sqrt{\mathrm{rg}\left(\frac{\mu+\tan\theta}{1-\mu\tan\theta}\right)}$$

Question

(a) Why a rider has to bend at a certain angle when moving round a bend.

(b) Derive the angle of inclination the rider makes with the horizontal when moving round a bend.

2. A bend of 200m radius on a level road is banked at the correct angle for a speed of $15ms^{-1}$. If a vehicle rounds the bend at $30ms^{-1}$, what is the minimum co-efficient of kinetic friction between the tyres and the road so that the vehicle will not skid.

Angle of banking

$$= \tan\theta = \frac{v^2}{rg} = \frac{15^2}{(200\times 9.8)}$$

$$\theta = 6.55^0$$

$$v^2 = rg \left(\frac{\mu + \tan\theta}{1-\mu\tan\theta}\right)$$

$$30^2 = 200\times 9.8 \left(\frac{\mu + \tan\ 6.55}{1\text{-}\ \mu\tan 6.55}\right)$$

$$900 = 1960 \left(\frac{\mu + 0.1148}{1\text{-}0.1148\mu}\right)$$

$$900 - 103.32\mu = 1960\mu + 225.008$$

$$2063.32\mu = 674.992$$

$$\mu = 0.327$$

2. A car travels round a bend in road which is a circular arc of radius 62.5m.

The road is banked at angle $\tan^{-1}\left(\frac{5}{12}\right)$ to the horizontal the coefficient of friction between the tyres of the car and the road surface is 0.4. Find

(i) the greatest speed at which the car can be driven round the bend without slipping.

(ii) The least speed at which this can happen.

(i) Maximum speed

$$v^2 = rg \left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$$

$$v^2 = 62.5 \times 9.8 \left(\frac{0.4 + 5/12}{1 - 0.4 \text{x} 5/12}\right)$$

$$v^2 = 612.5\left(\frac{\frac{49}{60}}{5/6}\right)$$

$$v^2 = 600.25$$

$$v^2 = 24.5ms^{-1}$$

(ii) Least speed

$$v^2 = rg\tan\theta$$

$$v^2 = 62.5 \times 9.8 \times 5/12$$

$$v^2 = 255.208$$

$$v = 15.98ms^{-1}$$

<u>Motion in a vertical circle</u>

This is an example of motion in a circle with non- uniform speed. The body will have a radial component of acceleration as well as a tangential component. Consider a particle of mass is attached to an inextensible string at point O, and projected from the lowest point P with a speed U so that it describes a vertical circle.

Consider a particle at point Q at subsequent time.

The tension T in the string is everywhere normal to the path of the particle and hence to its velocity V. the tension therefore does no work on the particle.

Energy at P, E_P is $E_p = ½mu^2$ ……………………..(1)

P is the reference for zero potential . Energy at Q in E_q is:-

$E_q = ½mv^2 + mgh$.

But $h = r\text{-}r\cos\Theta$

$Eq = ½ mv^2 + mgr (1-\cos\Theta)$ ………………….(2)

Centripetal force of the particle

$$T- mg\cos\Theta = \frac{mv^2}{r}$$

$Mv^2 = r (T-mg\cos\Theta)$ ………………………(3)

Substitute equation (3) into (2)

$E_q = ½ r (T- mg\cos\Theta) + mgr (1-\cos\Theta)$

Using conservation of mechanical energy

$E_q = E_p$.

$½r (T-mg\cos\Theta) + mgr (1-\cos\Theta) = ½mu^2$

$½r(T-mg\cos\Theta) = ½mu^2- mgr (1-\cos\Theta)$

$r(T-mg\cos\Theta) = mu^2 – 2mgr(1-\cos\Theta)$

$$T-mg\cos\Theta) = \frac{mu^2}{r} - 2mg(1-\cos\theta)$$

$$T = \frac{mu^2}{r} - 2mg(1-\cos\theta) + mg\cos\theta$$

$$T = \frac{mu^2}{r} + mg(2\cos\theta + \cos\theta - 2).$$

$$T = \frac{mu^2}{r} + mg(3\cos\theta - 2).$$

$$\text{OR } \mathrm{T} = \frac{\mathrm{mu}^2}{\mathrm{r}} - mg(2 - 3\cos\theta)$$

T is greater than zero when $\frac{mu^2}{r} + mg(\cos\theta - 2) > 0$

$$\frac{\mathrm{mu}^2}{\mathrm{r}} > mg(2 - 3\cos\theta)$$

$$u^2 > rg(2 - 3\cos\theta)$$

When $\vartheta = 90$

$$\mathrm{u}^2 > rg(2 - 3\cos 90)$$

$$u^2 > 2rg$$

Hence particle overshoots point 0' when u> $\sqrt{\ }$ 2rg

When ϴ = 180^0

$u^2 >$ rg (2-3cos 180)

$u^2 >$ 5rg

Hence particle reaches p' when U> $\sqrt{5rg}$

Therefore particle describes a circle when the initial speed with which you project from P is $u \geq \sqrt{5rg}$

Example

1. A cyclist rounds a curve of 30m radius on a road which is banked at an angle of 20^0 to the horizontal. If the co-efficient of sliding friction between the tires and the road is 0.5; find the greatest speed at which the cyclist can ride without skidding and find into inclination to the horizontal at this speed.

$$v^2 = rg\left(\frac{\mu + \tan\theta}{1-\mu\tan\theta}\right)$$

$$v^2 = (30\times 9.8)\left(\frac{0.5+\tan 20}{1-0.5\tan 20}\right)$$

$$v^2 = 294\left(\frac{0.1819}{0.818}\right)$$

$$v = 17.6ms^{-1}$$

$$\tan\theta = \frac{v^2}{rg} = \frac{17.6^2}{30x9.8}$$

$$\theta = 46.5°$$

4(b) A car goes round unbanked curve at $15ms^{-1}$the radius of the curve is 60m. Find the least co-efficient of kinetic friction that will allow the car to negotiate the curve without skidding.

$$\mu \geq \frac{v^2}{r}$$

$$\mu \geq \frac{v^2}{rg}$$

$$\mu \geq \frac{15^2}{(60\times 9.8)} = 0.38$$

Exercise

1. A stone of mass 0.5kg is attached to a string of length 0.5m which will break if the tension in it exceeds 20N. The stone is whirled in a vertical circle, the axis of rotation being at a vertical height of 1.0m above the ground. The angular speed is gradually increased until the string breaks.

(i) in what position is the string most likely to break?(vertically below point of suspension)

(ii) At what angular speed will the string break? ($7.7rads^{-1}$)

(iii) Find the position where the stone hits the ground when the string breaks.1.22m from point below point of suspension)

2. A car travels round a curved road bend banked at an angle of 22.6°. If the radius of curvature of the bend is 62.5m and the coefficient of friction between the tyres of the car and the road surface is 0.3. Calculate the maximum speed at which the car negotiates the bend without skidding. ($22.4ms^{-1}$)

GRAVITATION

Kepler's Law of Planetary Motion

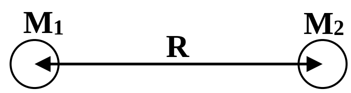
1. Planets revolve in elliptical orbits having the sun at one focus

2. Each planet revolve in such a way that the imaginary line joining it to the sun sweeps out equal areas in equal times

3. The squares of the periods of revolution of the planets are proportional to the cubes of their mean distances from the sun

Newton's Law of Gravitation

Every particle of matter attracts every other particle with a force which is proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.



$$F \propto \frac{M_1 M_2}{R^2}$$

Hence $F = \dfrac{G M_1 M_2}{R^2}$

Where G is a universal constant known as the Gravitational constant.

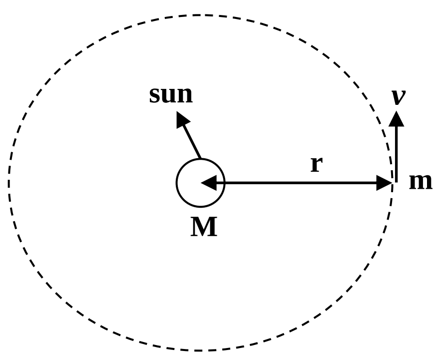
Units of G: Nm^2kg^{-2} or $m^3kg^{-1}s^{-2}$

Numerical value of G= $6.67 \times 10^{-11}\ Nm^2kg^{-2}$

Question: Show that the dimensions of G are $M^{-1} L^3 T^{-2}$

Proof of Kepler's 3rd law

Consider a planet of mass m moves with speed v in a circle of radius r round the sun of mass M.



Gravitational attraction of the sun for the planet, $F = \dfrac{G M m}{r^2}$

If this is centripetal force keeping the planet in orbit then

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

If T is the time for the planet to make one orbit

$$v = \frac{2\pi r}{T}$$

$$\frac{GMm}{r^2} = \frac{m}{r} \times \left(\frac{2\pi r}{T}\right)^2$$

$$GM = \frac{4\pi^2 r^3}{T^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Since $\frac{4\pi^2}{GM}$ is constant, then $T^2 \propto r^3$ which verifies Kepler's 3rd law.

Parking Orbit

A satellite launched with a speed such that its period equals that of the earth's rotation about its axis and is in the same sense as that of rotation of the earth is called the *Synchronous or Geostationary* satellite. To an observer on the earth's surface, such a satellite appears to be stationary. The orbit of the synchronous satellite is called a *Parking orbit*. Geostationary satellite can be used to relay TV signals and telephone. Messages from one point on the earth surface to other points. In this case a set of 3 synchronous satellites in a triangular array is used.

from

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

When the satellite is in a parking orbit, T = 24 hours = 24 x 3600s

$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

$M = 6 \times 10^{24}kg$

Hence $r = 4.23 \times 10^7 m$

Height above the earth for a parking orbit, $h = 4.23 \times 10^7$ - Radius of earth

But radius of earth = $6.4 \times 10^6 m$

Therefore, $h = 4.23 \times 10^7 - 6.4 \times 10^6 = 3.59 \times 10^7 m$

Variation of acceleration due to gravity

The acceleration due to gravity varies with both altitude and latitude

Variation of acceleration due to gravity with latitude

The acceleration due to gravity increases from $9.78ms^{-2}$ at the equator to $9.83ms^{-2}$ at the poles. The observed variation of g over the earth's surface is due to

(i) the effect of the earth's rotation

(ii) the non- spheroid of their earth

The effect of the earth's rotation: Because the earth rotates about it axis, its gravitational pull on the body on the equator has to provide a centripetal acceleration.

Effect of non- spheroid of the earth: The earth is not a sphere but an oblate spheroid whose equatorial radius exceeds polar radius by about 21.5km i.e. the body at the equator is slightly further away than at the poles. Hence acceleration at the poles is slightly exceeds the acceleration at the equator.

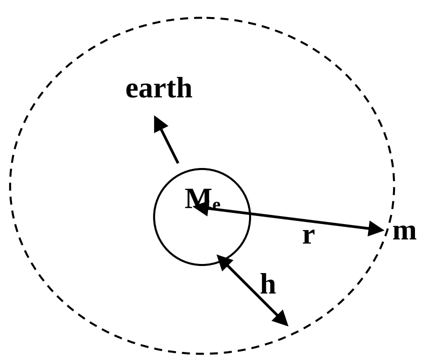
Variation of acceleration due to gravity with altitude

(i) At the earth's surface

$$Mg = \frac{GM_e m}{R_e}$$

$$g = \frac{GM_e}{R_e} \text{...(i)}$$

(ii) Above the earth's surface



If a body is at a point a distance r from the centre of the earth where $r > R_e$

Then $mg' = \frac{GMem}{r^2}$

g' is the acceleration due to gravity at the point a distance r from the centre of earth

$$g' = \frac{GMe}{r^2}$$

Hence $g' \propto \frac{1}{r^2}$

but from eqn (i) above, $GM_e = R_e^2 g$

$$g' = \frac{R_e^2 g}{r^2}$$

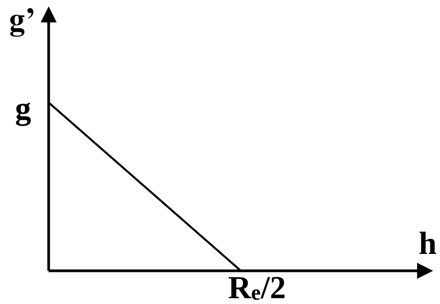
Also $r = h + R_e$

$$g' = \frac{R_e^2 g}{(h + R_e)^2} = g\,(1 + h/R_e)^{-2} = g\,(1 - 2h/R_e + 3h^2/R_e^2 + \ldots\ldots\ldots\ldots\ldots\ldots)$$

If h is smaller than R_e, then $(h/R_e)^2$ and higher powers can be ignored as they tend to 0.

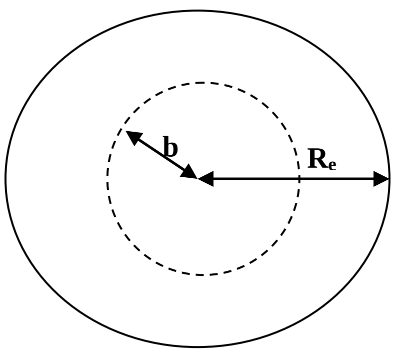
Therefore $g' = g(1 - 2h/R_e)$

A graph of g' against h appears as below:



(iii) *Inside the earth's surface*

Consider an object with mass m at a point which is a distance b from the earth's surface, where $b<R_e$. Let g'' be the acceleration due to gravity at this point and M_e' the effective mass of the earth at this point.



Assuming the earth to be a sphere of uniform density, ρ_e

$M_e = 4/3(\pi R_e^3)\rho_e$

$M_e' = 4/3(\pi b^3)\rho_e$

$M_e'/M_e = b^3/R_e^3$

$M_e' = (b^3/R_e^3)M_e$

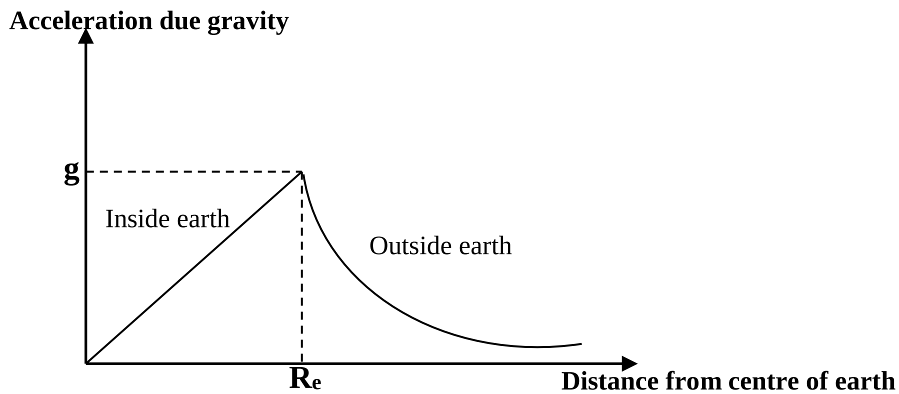
But $mg'' = \frac{G M_e' m}{b^2}$

$$g'' = \frac{G M_e'}{b^2} = \frac{G}{b^2} \times (b^3/R_e^3)M_e$$

$$g'' = \frac{GM_e b}{R_e^2}$$

Hence $g'' \propto b$

Graph showing variation of acceleration due to gravity with distance from centre of the earth.



Mechanical energy of a satellite

(i) Kinetic energy, E_k

Consider a satellite of mass m moving in a circular orbit of radius r. the centripetal force on the satellite is

$$\frac{G M_e m}{r^2} = \frac{mv^2}{r}$$

$$\frac{G M_e m}{r} = mv^2$$

The kinetic energy of the satellite, $E_k = \frac{1}{2}mv^2 = \frac{G M_e m}{2r}$

(ii) **Gravitational Potential energy, E_p**

The force of attraction between the earth and satellite of mass m at a distance *x* from the centre of the earth is

$$F = \frac{G M_e m}{x^2}$$

If the satellite is to move through ∂x towards the earth, the work done by the gravitational force is

$$\partial w = F\partial x = = \frac{G M_e m\partial x}{x^2}$$

If the satellite is moved from infinity to a point distance r from the centre of the earth, the work done by the gravitational force is

$$W = \int_{\infty}^{r} \frac{GM_e m}{x^2} dx = GM_e m\left[\frac{-1}{x}\right]_{\infty}^{r} = \frac{-GM_e m}{r}$$

Hence gravitational potential energy is the work done to move a body from infinity to a point in the gravitational field.

Therefore $E_p = \frac{-GM_e m}{r}$

Total mechanical energy $E_T = E_p + E_k = \frac{-GM_e m}{r} + \frac{GM_e m}{2r}$

$$= -\frac{GM_e m}{2r}$$

Note: The satellite has negative total energy hence it is a bound satellite.

Velocity of escape

Velocity of escape is the minimum vertical velocity with which the body must be projected from the earth so that it will never return to the earth.

The work done required for a body to escape = $-\frac{GM_e m}{R_e}$

If the body leaves the earth with speed v_e ad just escapes from its gravitational field

$$\tfrac{1}{2}mv_e^2 = \frac{GM_e m}{R_e}$$

Hence $v_e^2 = \frac{2GM_e}{R_e}$

$$v_e = \sqrt{\frac{2GM_e}{R_e}}$$

Exercise: Show that velocity of escape can be expressed as

$v_e = \sqrt{2gR_e}$

Effect of friction between a satellite and the atmosphere

Radius of orbit reduces, potential energy reduces, kinetic energy increases, velocity increases and mechanical energy decreases.

Examples:

1. A satellite of mass 100kg is inn a circular orbit at a height of 3.59×10^7m above the earth's surface. Find the mechanical energy of the satellite.(Mass of earth = 6×10^{24}kg, radius of earth = 6.4×10^6m)

Mechanical energy = $-\dfrac{GM_e m}{2r}$

$r = 3.59 \times 10^7 + R_e = 3.59 \times 10^7 + 6.4 \times 10^6 m = 4.23 \times 10^7 m$

Where $R_e = 6.4 \times 10^6$m, the radius of the earth.

$M_e = 6 \times 10^{24}$kg

Mechanical energy = $-\dfrac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 100}{2 \times 4.23 \times 10^7} = -4.71 \times 10^8$ joules

2. A satellite of mass 250kg makes a circular equatorial orbit at a distance 500km above the earth's surface. Find

(i) the radius of the orbit

(ii) the period

(iii) the total energy of the satellite

(i) radius $r = 500 \times 10^3 + 6.4 \times 10^6 = 6.9 \times 10^6$m

(ii) $T^2 = \left(\dfrac{4\pi^2}{GM}\right) r^3$

Where $G = 6.67 \times 10^{-11}$ Nm^2kg^{-2}

$M = 6 \times 10^{24}$kg

$T^2 = \dfrac{4\pi^2 x(6.9x106)^3}{6.67 \times 10^{-11} \times 6 \times 10^{24}}$

Hence $T = 5.69 \times 10^3$s

(iii) Total energy = $-\dfrac{GM_e m}{2r} = -\dfrac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 250}{2 \times 6.9 \times 10^6} = -7.25 \times 10^9$J

Exercise

1. A mass is released from a point at a distance of 10R from the centre of the earth, where R is the radius of the earth. Find the speed of the mass at a point a distance of 7R from the centre of the earth. (Assume R = 6.4×10^6m)

2. Calculate the ratio of mass of the sun to that of the earth, given that the moon moves round the earth in a circular orbit of radius 4.0×10^5km with a period of 27.3days, and the orbital radius of the earth round the sun is 1.5×10^8km and its period is 365days. (2.95×10^5)

3. Calculate the ratio of acceleration due to gravity on the surface of mercury to that on the surface of the earth given that the radius of mercury is 0.38 times that of the earth and the mean density of mercury is 0.68 times that of the earth (0.2584)

Simple Harmonic Motion (S.H.M)

It a special type of periodic motion in which the acceleration of the body along the path of the body is directed towards a fixed point in the line of motion and is proportional to the displacement of the body from the fixed point.

Characteristic of a body describing Simple harmonic motion

- Motion is periodic
- Acceleration of the body is towards a fixed point
- Acceleration of the body is directly proportional to the distance from the fixed point
- Mechanical energy is conserved.

Equation of simple harmonic motion

Acceleration, $a = -\omega^2 x$

Where ω is angular velocity, x is displacement from fixed point.

Or $$a = \frac{d^2x}{dt^2} = -\omega^2 x$$

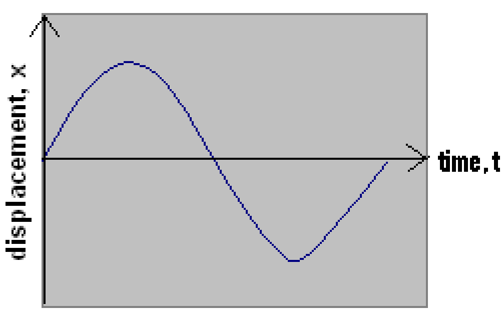
The solution of the above differential equation is

$X = A\cos\omega t$ or $X = A\sin\omega t$

Where A is the maximum displacement of the body called <u>Amplitude</u>.

For $X = A\sin\omega t$ the curve is as below:

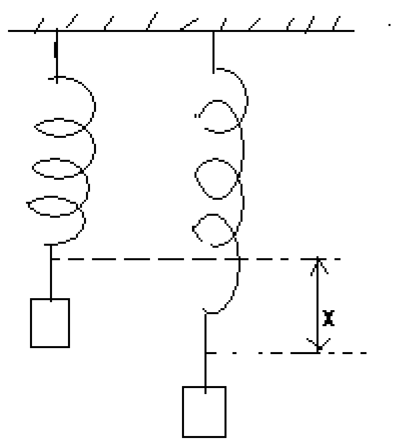
A graph of displacement against time for S.H.M



In general $X = A\sin(\omega t + \Phi)$ Where Φ is the phase angle.

Examples of Simple Harmonic Motion

(i)*Vertical Spiral Spring or Elastic thread*



Consider a body of mass m suspended from a spiral spring of force constant, k, as shown in the diagram. In that case the body will be at equilibrium.

At equilibrium, $T = mg$

But $T = ke$ (Form Hook's Law)

Where e is the extension in the spring at equilibrium and k is the force constant of the spring.

Hence $ke = mg$ ………………………………….(i)

When the mass is pulled through a distance x then released, the resultant upward force on the mass is

$F = T' - mg$

But $T' = k(e + x)$

$F = k(e + x) - mg$

But from (i) $ke = mg$

$F = k(e + x) - ke$

$F = kx$

From Newton's 2nd law, $ma = F$

$ma = -kx$

$$a = -\left(\frac{k}{m}\right)x$$

The above equation is in the form $a = -\omega^2 x$

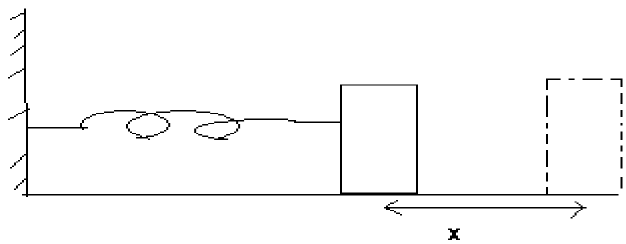
Where $\omega^2 = \left(\frac{k}{m}\right)$

Question: Prove that the period T is given by**:** T **=** $2\pi\sqrt{\frac{m}{k}}$

(ii) *Horizontal* Spiral *Spring*

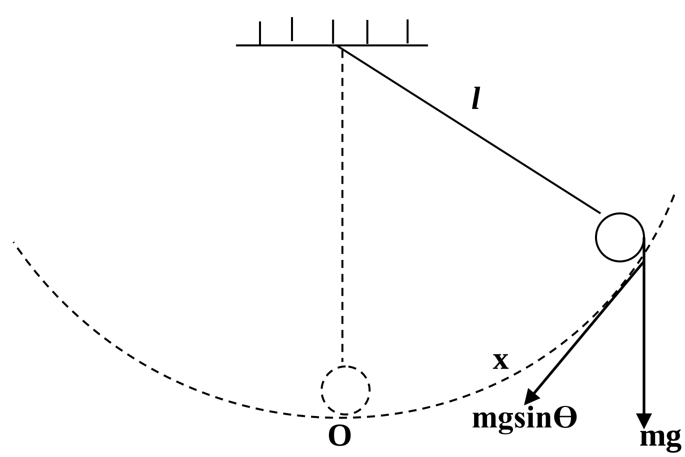
Consider a horizontal spring of force constant k. One end of the spring is fixed and the other end attached to a body of mass m resting on a smooth surface.

If the body is displaced through a distance x



(iii) Simple Pendulum

Suppose a body of mass m attached to a string is displaced through a small angle ϴ and then released. The resultant force on the body towards O is mgsinϴ.



By Newton's 2nd law

$ma = -mg\sin\Theta$

$a = -g\sin\Theta$

If Θ is small and measured in radians $\Theta \cong \sin\Theta = \frac{x}{l}$

$a \cong -g\Theta = g\frac{x}{l}$

Which is in the form $a = -\omega^2 x$

Where $\omega = \sqrt{\frac{l}{g}}$

Hence $T = 2\pi\sqrt{\frac{l}{g}}$

Example: A simple pendulum has a period of 4.2s. When the length is shortened by 1m, the period is 3.7s. Use these measurements to determine the acceleration due to gravity and the original length of the pendulum.

$T = 2\pi\sqrt{\frac{l}{g}}$

$4.2^2\, g = 4\pi^2 x l$……..*(1)*

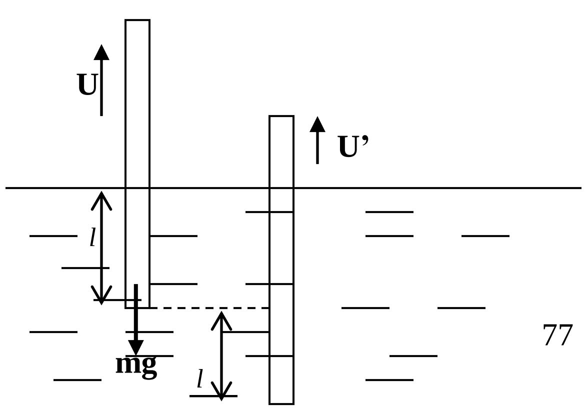
$3.72g = 4\pi 2(l - 1)$……….(2)

Solving the above two equations, you get

$L =$, g =

(iv) *A Floating cylinder*

Consider a cylinder of mass m, floating vertically in a liquid of density ρ to a depth l.



In equilibrium, mg = U where U is upthrust

But U = Al ρg

mg = Alρg ………………………………(i)

m = Alρ

A is the cross sectional area of the cylinder

Suppose the cylinder is given a small vertical displacement x and released, the net force on the cylinder is U' – mg.

But U' = A (l +x)ρg

Net force = A (l +x)ρg – mg

From Newton's 2nd law; ma = - A (l +x)ρg – mg

From equation (i) mg = Alρg

Therefore ma = - Axρg

$$a = -\frac{(A\rho g)x}{m}$$

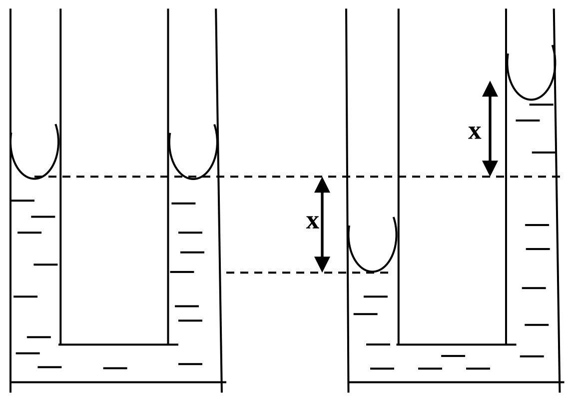
But $a = -\omega^2 x$

Where $\omega^2 = \frac{A\rho g}{m}$

And period $T = 2\pi\sqrt{\frac{A\rho g}{m}} = 2\pi\sqrt{\frac{l}{g}}$

(v) *Oscillation of a liquid in a U – tube*

Consider a liquid column of length *l* at rest in a U – tube of cross section area A. Suppose the liquid is displaced by a small distance and then released.



Consider the instant when the meniscus a distance x from their equilibrium position. The restoring force of the liquid = 2xAρg, where ρ is the density of the liquid.

Using Newton's 2^{nd} law,

$$ma = -2xA\rho g$$

$$a = \frac{-(2xA\rho g)}{m} = -\frac{(2A\rho g)}{m}x$$

Hence $\omega^2 = -\frac{(2A\rho g)}{m}$

Period $T = 2\pi\sqrt{\frac{l}{2g}}$

Velocity of a body executing Simple harmonic Motion

Consider the displacement of a body executing Simple harmonic motion to be given by

$X = A\sin(\omega t + \Phi)$

Velocity, $v = dx/dt = A\omega\cos(\omega t + \Phi)$

$\sin(\omega t + \Phi) = X/A$

$\text{Cos}(\omega t + \Phi) = \dfrac{\sqrt{A^2 - x^2}}{A}$

Hence $v = A\omega \dfrac{\sqrt{A^2 - x^2}}{A} = \omega\sqrt{A^2 - x^2}$

When x = 0, V is maximum

$v_{max} = \omega A$

when X= A, v = 0

Kinetic energy and potential energy of vibrating object

Kinetic enrgy, E_k

Velocity $v = \omega\sqrt{A^2 - x^2}$

Kinetic energy $E_k = ½mv^2 = ½m\omega^2(A^2-x^2)$

$$E_k = ½m\omega^2(A^2-x^2)$$

For a spring of force constant, k ; $\omega^2 = \dfrac{k}{m}$

$$k = \omega^2 m$$

$$E_k = ½k(A^2-x^2)$$

Potential energy, E_p

Work done against the restoring force is the potential energy

$F = m\omega^2 r$

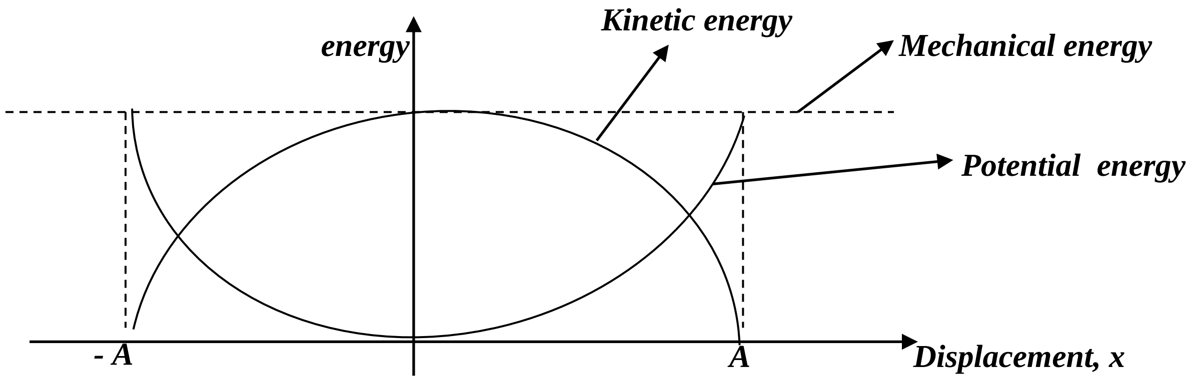
Therefore, $E_p = \int_0^x Fdr = \int_0^x m\omega^2 r dr = ½m\omega^2x^2$

For a vibrating spring, $E_p = ½kx^2$

Total mechanical energy $E_T = E_k + E_p = ½m\omega^2(A^2-x^2) + ½m\omega^2x^2 = ½m\omega^2A^2$

$$E_T = ½m\omega^2A^2$$

Note total energy of a vibrating object (a particle undergoing S.H.M) is constant and is directly proportional to the square of the amplitude. Hence mechanical energy is conserved in S.H.M.



Examples:

1. A light spiral spring is loaded with a mass of 50g and it extends by 10cm. Calculate the period of small vertical oscillations

Using $T = 2\pi\sqrt{\frac{m}{k}}$, but mg = ke

K = mg/e = 0.05x9.81/0.1 = $4.905Nm^{-1}$

Hence $T = 2\pi\sqrt{\frac{0.05}{4.905}}$ = 0.63s

2. A body of mass 0.1kg hangs from a long spiral spring. When pulled down 10cm below its equilibrium point A, and released, it vibrates with S.H.M with a period of 2s.

(i) What is the velocity as it passes through A?

(ii) What is its acceleration when it is 5cm above A.

Solution

(i) $v = \omega A$, where $A = 0.1m$, $\omega = 2\pi/T$, but $T = 2s$

$\omega = 2\pi/2 = \pi$ $rads^{-1}$

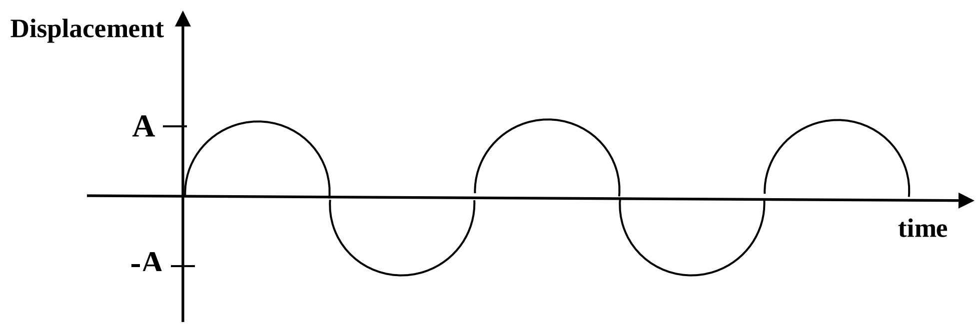
$V = \pi \times 0.1 = 0.314ms^{-1}$

(ii) $a = -\omega^2 x = \pi^2 \times 0.05 = 0.5ms^{-2}$

Types of oscillations

(i) Free oscillations:

Free oscillations occur in the absence of any dissipative forces like air resistance, friction and viscous drag. The amplitude and total mechanical energy remains constant and the system oscillates indefinitely with a period T (the natural period of vibration of the system)



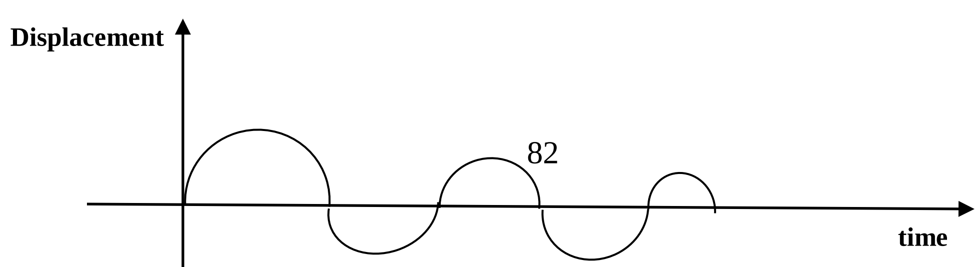
A is amplitude

e.g A simple pendulum will undergo free oscillation in a vacuum.

(ii) Damped oscillations

These are oscillations where the system loses energy to the surrounding due to the dissipative forces. The amplitude reduces with time and oscillations eventually die out. Damped oscillations can be grouped into under damped, critically damped and over damped oscillations.

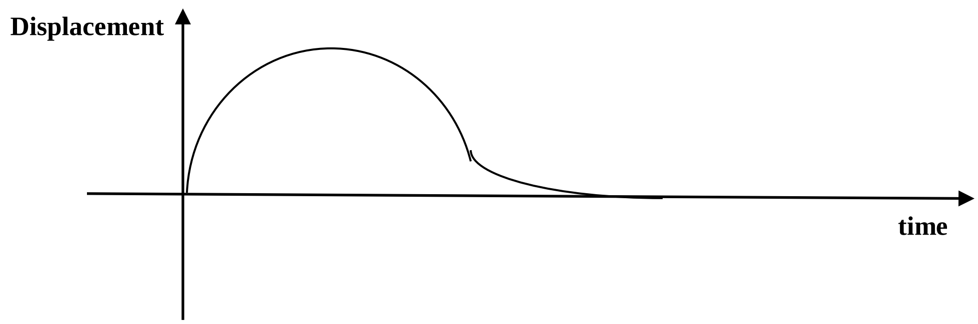
- Under – damped oscillations



The system actually oscillates but gradually dies out due to the dissipative forces. The amplitude of oscillation decreases with time. Examples are a simple pendulum in air, horizontal spring moving over a surface of little roughness.

- Critically damped oscillations

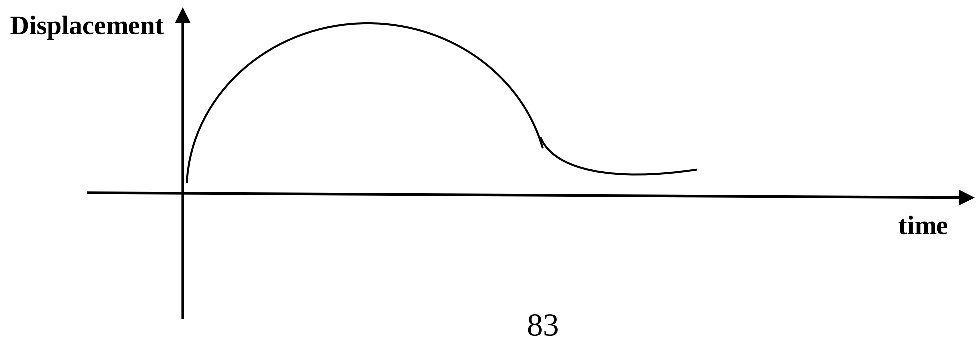
The system does not oscillate when displaced, but returns to the equilibrium position in the minimum possible time



Examples shock absorbers in cars stops the car to oscillate after passing over the hump, toilet doors are critically damped so that they close very quickly.

- Over damped oscillations

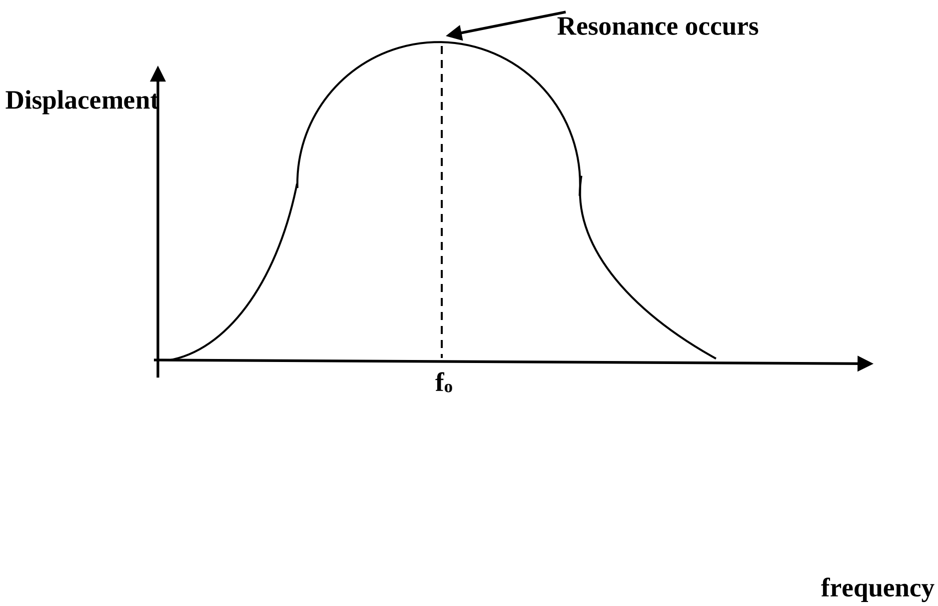
The system does not oscillate but takes a long time to return to the equilibrium position.



Examples: a horizontal spring moving over a very rough surface, a metal cylinder attached to a vertical spring and made to move in a very viscous liquid.

Forced oscillations

These are vibrations where the system is subjected to an external periodic force thus setting the system to oscillate indefinitely. When the periodic force has the same frequency of oscillation as the system, resonance occurs. Examples of forced oscillations are; the oscillation of a diving board, the oscillations of the earth quake and the oscillations of the air column in musical pipe instruments e.t.c



f_o is the fundamental frequency

Exercise:

1. The pendulum of length 130cm has a periodic time T_1. A bob now pulled a side and made to move as a conical pendulum in a horizontal circle of radius 50cm. the period of rotation is T_2. Find the ratio
of T_1: T_2 (1.04)

2. A spring gives a displacement of 5cm for a load of 500g. Find the maximum displacement produced when a mass of 80g is dropped from a height of 10cm onto a light pan attached to the spring.
($5x10^{-2}$m)

3. A small mass rests on a horizontal platform which vibrates vertically in a simple harmonic motion with a period of 0.50s. Find the maximum amplitude of the motion which will allow the mass to remain in contact with the platform throughout the motion.($6.3x10^{-2}$m)

4. A mass of 0.1kg suspended from a spring of force constant 24.5Nm^{-1} is pulled vertically downwards through a distance of 5.0cm and released. Find the
   (i) period of oscillation (0.4s)
   (ii) position of the mass 0.3s after release(0m)

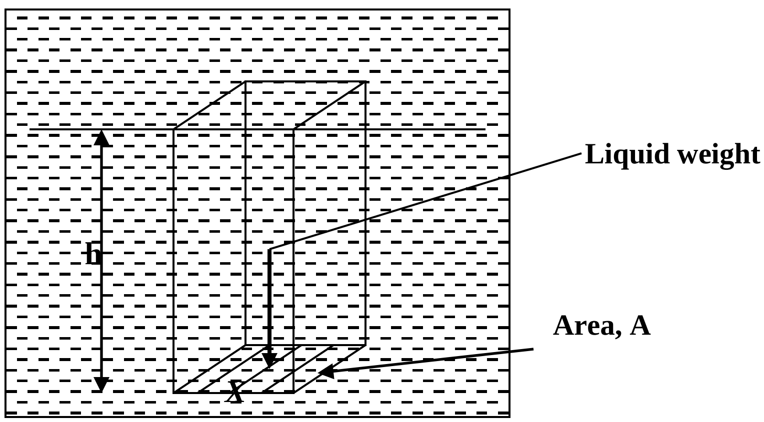
5. A uniform cylindrical rod of length 8cm, cross sectional area 0.02m^2 and density 900kgm^{-3} floats vertically in a liquid of density 1000kgm^{-3}. The rod is depressed through a distance of 0.5cm and the released.
   i) Show that the rod performs simple harmonic motion
   ii) Find the frequency of the resultant oscillations (1.86Hz)
   iii) Find the velocity of the rod when it is a distance of 0.4cm above the equilibrium position. (0.035ms^{-1})

Pressure

The pressure at a point in a fluid is the force per unit area acting normal to an infinitesimal area taken about the point. The units of pressure Nm^{-2} or Pascal (Pa). The pressure in a column of fluid increase with depth. At a given point in a liquid, pressure acts in any direction hence it is a scalar quantity.

Formula for pressure in liquids

Sippose that a horizontal plate X of area A is placed at a depth h below the liquid surface. By drawing vertica lines from the points on the perimeter of X, we can see that the force on X due to the liquid is equal to the weight of the liquid of height h and uniformm cross section A.



Since the volume of this liquid is Ah, the mass of the liquid = Ahρ.

The weight = Ahρg , where g is acceleration due to gravity.

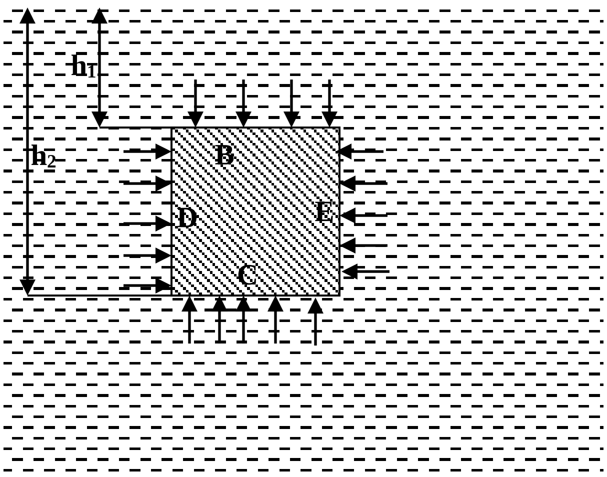
Therefore the pressure, P = $\frac{force}{area} = \frac{Ah\rho g}{A} = h\rho g$

Archimedes principles

When an object is immersed in a fluid, it experiences an upward force called up thrust which is equal to the weight of the fluid displaced.

Proof of Archimedes principle

Consider a uniform solid cylinder of length L, cross sectional area A. Suppose the cylinder is submerged in a liquid of density P, so that its face is a depth h, below the surface of the liquid.



Consider a solid immersed in a liquid, the pressure on the lower surface C is greater than on the upper surface B, since the pressure at the greater depth h_2 is more than at h_1. The net horizontal force is zero.

The upward force on C = $h_2\rho gA$, where ρ is the liquid density.

The down wad force on side B = $h_1\rho gA$.

Thus resultant force on solid = upward force (upthrust) = $(h_2 - h_1)\rho gA$.

But $(h_2 - h_1)A$ = volume, V, of solid,

Therefore upthrust = $V\rho g = mg$, where $m = V\rho$.

Therefore upthrust = weight of liquid displaced.

Measurement of density or relative density using Archimedes' principle.

For a solid, weigh the mass of solid in air say, m_o. Then weigh its mass when totally immersed in water say, m_1.

Then upthrust = $(m_o - m_1)$ g = weight of water displaced.

Therefore relative density = $\frac{m_0}{m_0 - m_1}$

Density of solid = $\frac{m_0}{m_0 - m_1}$ x density of water

***For a liquid*.**

Weigh the mass of solid in air say, m_0, then weigh it when totally immersed in the liquid whose density is required say m_1 and finally weigh it when totally immersed in water say m_2.

Relative density = <u>upthrust in liquid</u>
upthrust in water

$$= \frac{m_0 - m_1}{m_0 - m_2}$$

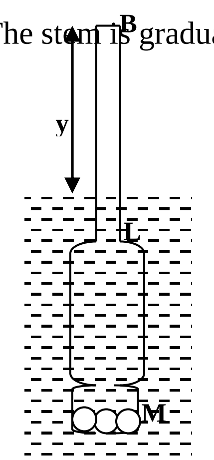
And density = $\frac{m_0 - m_1}{m_0 - m_2}$ x density of water.

<u>Law of flotation</u>

A floating body displaces its own weight of the fluid in which it floats.

THE HYDROMETER

This is a device for comparing densities of liquids. It consists of a uniform stem having a loaded bulb at the bottom. The stem is graduated in which it is placed.



Practical hydrometers have a weighted end M for stability, a wide bulb tp produce sufficient upthrust to counterbalance the weight, and a narrow stem BL for sensitivity. If V is the whole volume of the hydrometer, a is the area of the stem and y is the length not immersed in a liquid of density,ρ, then upthrust = weight of liquid displaced

= (V – ay)ρ = w, where w is the weight of the hydrometer.

Examples.

1. A cube of rubber, volume $10^{-3}m^3$, floats with half of its volume submerged in a liquid of density $1200kgm^{-3}$. Find the depth to which the cube would be submerged in a liquid of density $1000kgm^{-3}$.

$L = 10^{-1}m$

Volume = $L^3 = 10^{-3}m^3$

When immersed in liquid of density $1200kgm^{-3}$ volume of liquid displaced = $\frac{1}{2}x10^{-3}$ = $5x10^{-4}m^3$.

$\therefore$ mass of liquid displaced = $5x10^{-4}$ x 1200 = $6x10^{-1}kg$

hence using law of flotation, mass of body = $6x10^{-1}kg$

when immersed in liquid of density 1000 kgm^{-3}

mass of liquid displaced = $6x10^{-1}kg$.

volume of liquid displaced = $m/d = 6 \times 10^{-4} m^3$. If h is the depth

$$l^2 h = 6 \times 10^{-4}$$
$$10^{-2} h = 6 \times 10^{-4}$$
$$h = 6 \times 10^{-2} m.$$

2. A solid weight 237.5g in air and 12.5g when totally immersed in a liquid of density $0.9gcm^{-3}$. Calculate (a) Density of solid (b)

The density of the liquid in which the solid would float with $\frac{1}{5}$ of its volume exposed above the liquid surface.

When immersed in liquid of density $0.9gcm^{-3}$, Loss in mass = 237.5 – 12-5= 225g.

Therefore mass of liquid displaced = 225g

Volume of liquid displaced = $^{225}\!/_{0.9}$

$= 250cm^3$

Hence volume of the body = $250cm^3$

$\therefore$ Density of solid = $^m\!/_v = \frac{237.5}{250} = 0.95gcm^{-3}$

b) Volume of liquid displaced = $\frac{4}{5} \times 250 = 200cm^3$

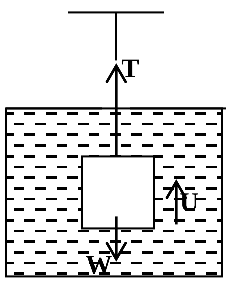
Mass of the liquid displaced = 200ρ

Using law of flotation, Mass of liquid displaced = mass of body =

$200\ \rho = 237.5$

$\rho = 1.187gcm^{-3}$

3. A string supports a solid iron object of mass 180g, totally immersed in a liquid of density $800kgm^{-3}$. Calculate the tension in the string if the density of iron is $8000kgm^{-3}$.



Weight of body, W = mg = 0.18x9.81 = 1.764N

Volume of object = $^m\!/_d = \frac{0.18}{8000} = \text{volume of liquid displaced}$

$= 2.25 \times 10^{-5}$

Upthrust, U = Alpg = $2.25x10^{-5}x800x9.8$ = 0.176N

Hence tension, T = W –U = 1.764 – 0.176 = 1.5836N

Exercise:

1. A piece of metal of mass $2.60x\ 10^{-3}kg$ and density $8.4\ x\ 10^3\ kgm^{-3}$ is attached to a

block of wax of mass 1.0 x 10^{-2}kg and density 9.2 x 10^{2} kgm^{-3}. When the system is placed in a liquid it floats with wax just submerged. Find the density of the liquid.(1.13x10^{3} kgm^{-3})

2. A block of mass 0.10kg is suspended from a spring balance. When the block is immersed in water of density 1.0x$10^{3}$$kgm^{-3}$, the spring balance reads 0.63N. When the block is immersed in a liquid of unknown density, the spring balance reads 0.70N. Find
(i) the density of the solid (2795kgm^{-3})
(ii) the density of the liquid (800 kgm^{-3})

3. A string supports a metal block of 2kg which is completely immersed in a liquid of density 8.8x$10^{2}$$kgm^{-3}$. If the density of the metal is 9x$10^{3}$$kgm^{-3}$, calculate the tension in the string. (17.7N)

4. A hydrometer floats with 6.0cm of its graduated stem unimmersed, and in oil of relative density 0.8 with 4.0cm of the stem unimmersed. What is the length of the stem unimmersed when the hydrometer is placed in a liquid of relative density 0.9?(5.1cm)

5. A block of volume 1000cm^{3}mfloats half – immersed in a liquid of relative density 1.2. Calculate the volume of brass, relative density 8.7 which must be attached to the wood in order that the combination just floats in a liquid of relative density 2.2. (246cm^{3})

6. A hydrometer consists of a spherical bulb and a cylindrical stem of cross-sectional area 0.4cm^{2}. The total volume of the bulb and stem is 13.2cm^{3}. When immersed in water, the hydrometer floats with 8.0cm of the stem above the water surface in alcohol it floats with 1.0 cm of the stem above the surface. Calculate the density of the alcohol.(0.78gcm^{-3})

SURFACE TENSION

Some observation due to surface tension

1. A drop of water, on closing a tap remained dinging on the tap, as if the water was held in a bag.

2. A thin needle can be made to float on the surface though it is denser than water.

3. Mercury gathers in small spherical drops when poured on a smooth surface

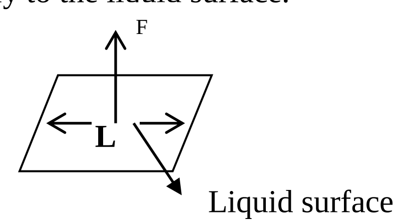
4. When a capillary tube is dipped in water, water is seen rising up in a tube.

5. Insects can walk on the water surface

All the above observations show that a liquid surface behaves as if it was or it is in a state of tension. The phenomenon is called surface tension.

Surface Tension or Co-efficient of surface tension (γ)

This is the force per unit length acting in a liquid surface at right angle to an imagining drawn tangentially to the liquid surface.



$\therefore \gamma = F/L$

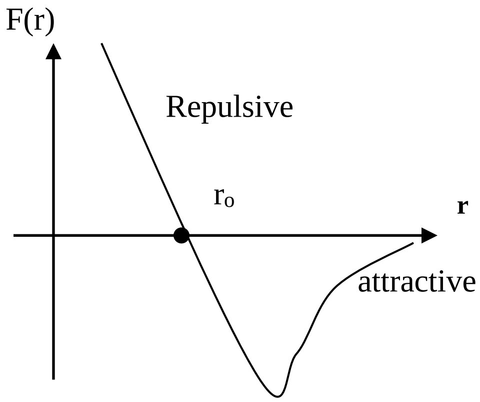
The units of γ are Nm^{-1}

$[\gamma] = MLT^{-2}L^{-1}$

$= MT^{-2}$

Molecular Theory of Surface Tension

The force F(r) between two molecules of a liquid varies with their separation r as shown below

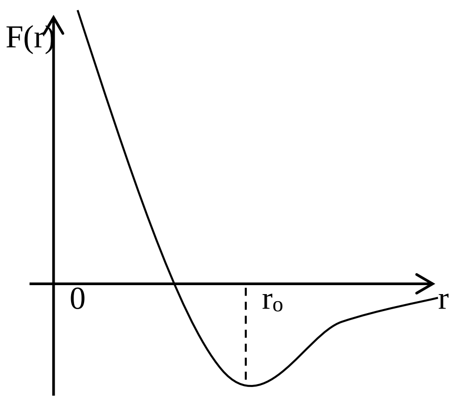


At the average equilibrium separation, $r_{0,}$ $F(r) = 0$

For $r > r_o$ = the force is attractive.

For $r < r_o$ = the force is repulsive.

The corresponding potential energy variation with molecular separation is shown below

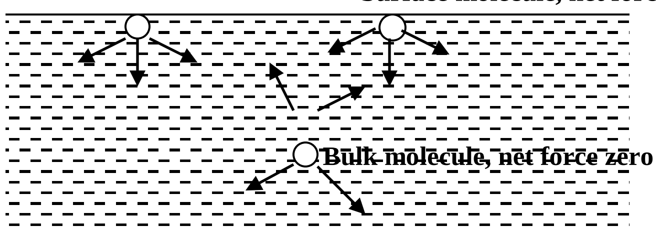


- The molecule within the body of the liquid (bulk molecule) is attracted equally by neighbors in all directions, hence the force on the bulk molecule is zero, so the intermolecular separation for bulk molecules is r_o.

- For a surface molecule, there is a net inward force because there are no molecules above the surface. Hence to bring a molecule from inside the liquid.

- To the surface, work must be done against the inward attractive force, hence a molecule in the surface of the liquid has a greater potential energy than a molecule in bulk. The potential energy stored in the surface is called free surface energy.

- Molecules at the surface have their separation $r > r_o$ The attractive forces experienced by surface molecules due to their neighbours put them in a state of tension and the liquid surface behaves as a stretched skin.

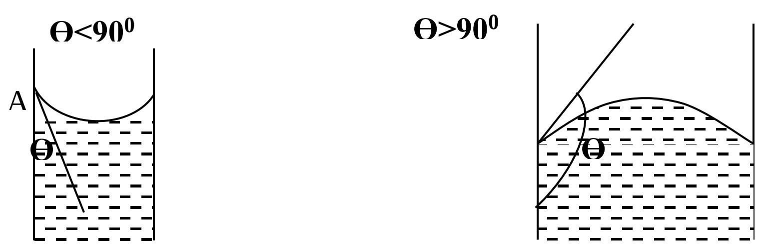


Surface energy and shape of a drop of a liquid

All systems arrange themselves in such a way that they have the minimum possible potential energy. The number of molecules that resides in the surface has to be minimum, and to minimize the number of molecules on the surface, the surface area must be reduced, hence liquid surface contract to the smallest possible area. So free liquid drops are spherical for any given volume because it is the shape which gives the minimum surface area. A large drop flattens out in order to minimize the gravitational potential energy which tends to exceed the surface energy. Due to its large weight, gravitational force distorts the spherical shape of large droplets however a small drop takes on a spherical shape to minimize the surface energy, which to be greater than gravitational potential energy. Therefore the gravitational force can not distort the spherical shape due to very small mass of tiny droplets.

Angle of contact

The angle between the solid surface and the tangent to the liquid surface at the point of intersection with the solid surface as measured through the liquid.



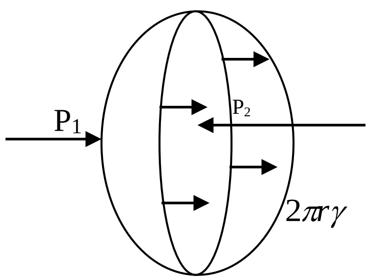
θ = angle of contact

A= point of intersection with solid surface

A liquid makes an acute angle of contact with the solid surface if the adhesive forces between the liquid and solid molecules are greater the cohesive forces between the liquid molecules themselves. The angle of contact is zero on a clean glass for pure water. If a liquid makes an acute angle of contact, it is said to wet the solid surface. A liquid makes an obtuse angle of contact with the solid surface if the cohesive forces between the liquid molecules themselves are greater than the adhesive forces between the solid and liquid molecules. Such a liquid is said not to wet the solid surface. The angle of contact of mercury on a glassurface is 140^0. Addition of detergent to a liquid reduces the angle of contact and therefore helps in washing.

Excess pressure inside an air bubble

Consider the equilibrium of one half of an air bubble of radius r, in a liquid of surface tension γ



This half of the bubble is in equilibrium under the action of force F_1 which is due to pressure P_1, F_2 which is due to the pressure p_2 and force F

P_1= pressure outside the bubble

P_2 = pressure inside the bubble

For equilibrium, $F_1+F = F_2$

$F_1 = P_1.\pi r,$ $\quad$ $F_2 = p_2.\pi r^2,$ $\quad$ $F = 2\pi r\gamma$

Hence $P_1\pi r^2 + 2\pi r\gamma = p_2\pi r^2$

But $(p_2 - p_2)r = 2\gamma$

$$p_2 - p_1 = \frac{2\gamma}{r} \quad \text{(Excess press for air bubble)}$$

Excess pressure inside a soap bubble

For a soap bubble, it has two surfaces

$F = 2.2\pi r\gamma$

For equilibriuim

$F_1 + F = F_2$

But $F_1 = P_1\pi r$ $\quad$ $F = 4\pi r\gamma$ $\quad$ $F_2 = p_2\pi r^2$

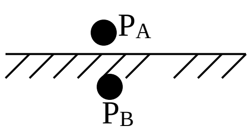
$$p_1\pi r^2 + 4\pi r\gamma = p_2\pi r^2$$

$$(p_2 - p_1)r = 4\gamma$$

$$(p_2 - p_1) = \frac{4\gamma}{r}$$

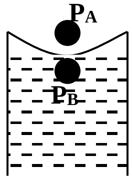
Note. The pressure on the concave side of a liquid surface is always greater than that on a convex side e.g.

Flat surface $\quad$ $P_A = P_B$



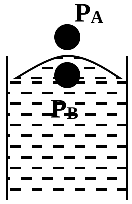
Hence excess is equal o zero on a flat surface.

Concave meniscus



$P_A - P_B = \frac{2\gamma}{r}$ where I is the radius of the meniscus

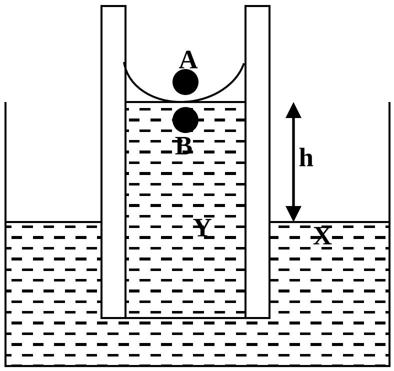
Convex meniscus



$$P_B - P_A = \frac{2\gamma}{r}$$

Capillary Rise

Consider the care of a liquid wets glass.



Pressure at X = pressure at Y = P_o (atmospheric pressure)

But $P_A - P_B = \frac{2\gamma}{r}$ r is radius of meniscus

$P_B = P_A - \frac{2\gamma}{r}$

$$P_y = P_B + h\rho g$$

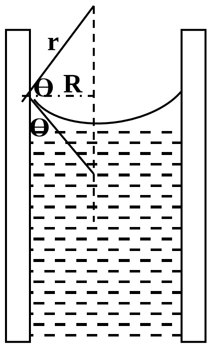
$$p_y = p_A - \frac{2\gamma}{r} + h\rho g$$

But $p_0 = p_0 - \frac{2\gamma}{r} + h\rho g$

$$hpg = \frac{2\gamma}{r}.$$

$\therefore h = \frac{2\gamma}{r\rho g}$ height whi ch liqiud rises

The radius of curvature of the meniscus is related to the radius of the capillary and angle of contact as shown



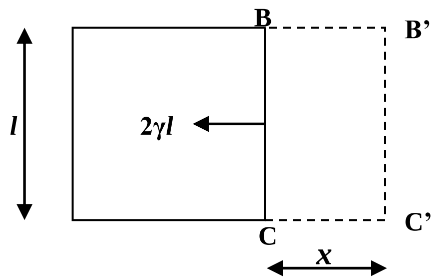
$$r = \frac{R}{\cos\theta} \qquad \text{Hence } h = \frac{2\gamma\cos\theta}{R\rho g}$$

Effects of temperature on surface tension

When the temperature of a liquid is raised, the mean kinetic energy of the molecules of the liquid raises on the average of the force of attraction between the molecules decreases since the molecules spend less time in the neighbourhood of the given molecules as a result the intermolecular separation rises hence surface tension of the liquid decreases with rising temperature.

Relationship between surface energy and surface tension

Consider a liquid stretched on a rectangular metal frame



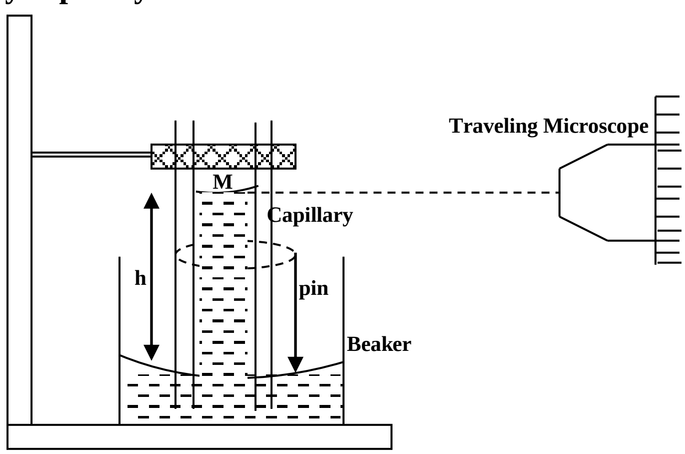
Suppose a film is stretched isothermally (at constant temperature) so that the edge BC moves through a distance ***x*** to B'C'. The work done to stretch the film = $F_0\boldsymbol{x}$

But $F = 2\gamma l$ (the film has 2 surfaces)
work done $= 2\gamma l x$
increase in area $= 2lx$
Work done to increase a unit area $= \dfrac{2\gamma l x}{2lx} = \gamma$

Hence surface tension can also be defined as the work done to increase surface area of a liquid by $1m^2$ under isothermal condition.

<u>Measurement of surface Tension</u>

By capillary rise method

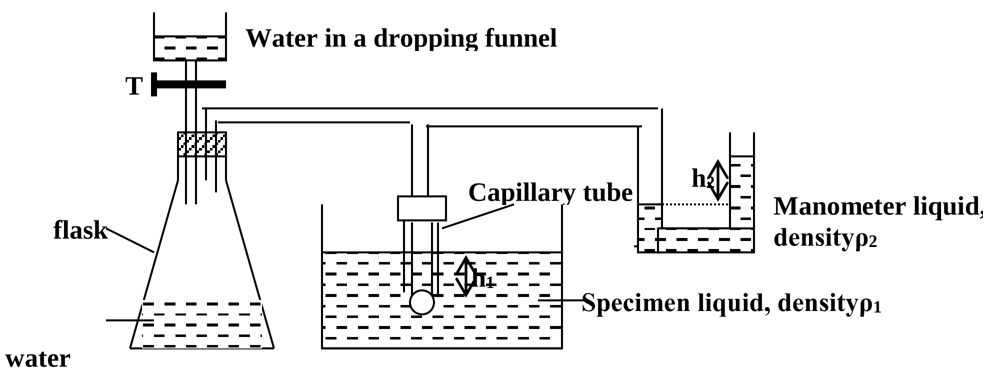


A pin is attached to the capillary tube with its tip just touching the liquid in the beaker. A traveling microscope is focused on the meniscus M. The reading S_1, on the scale is recorded. The beaker is carefully removed and the traveling microscope is focused on the tip of the pin P. The reading S_2 on the scale is recorded.

The capillary rise h = $|S_2 - S_1|$.

The radius, r of the capillary tube is determined measuring its diameter by using a traveling microscope. The angle θ of contact is measured and since the density, ρ of the liquid is known, surface tension can be calculated from; $\gamma = \frac{hr\rho g}{2\cos\theta}$

Jaeger's method



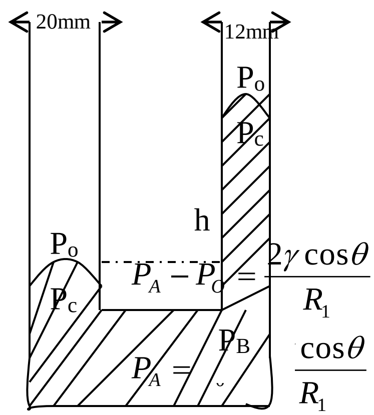
The pressure in the flask is increased gradually by allowing drops to fall down the funnel. Bubbles formed at the tip of the capillary tube dipping in the specimen liquid are observed. When the bubble has grown to a hemispherical shape, the tap T is closed and the reading h2 on the manometer is recorded. The depth, h1 of the end of the capillary tube below the specimen is recorded. Using $\frac{2\gamma\cos\theta}{a} + h_1\rho_1 g = h_2\rho_2 g$

$$\gamma = \frac{(h_2\rho_2 - h_1\rho_1)ga}{2\cos\theta}$$

The radius, a of the capillary tube is determined measuring its diameter by using a traveling microscope. The angle θ of contact is measured and since the density, ρ_1, ρ_2 of the liquids are known, then γ can be calculated.

Examples

1) Mercury is poured into a glass U- tube with vertical limbs of diameters 20mm and 12.00mm respectively. If the angle of contact between mercury and glass is 140^0 and the surface tension of mercury is 0.152 Nm^{-2}. Calculate the difference in the levels of mercury. (Density of mercury = 1.35 x 10^4 kgm^{-3}).



$$p_B = P_A + hpg$$

$$P_B = P_0 + \frac{2\gamma\cos\theta}{R_1} + hpg.........(i)$$

$$P_C - P_0 = \frac{2\gamma\cos\theta}{R_2}$$

$$P_C = P_0 + \frac{2\gamma\cos\theta}{R_2}.....................(ii)$$

$$but P_B = P_c$$

$$hence P_o + \frac{2\gamma\cos\theta}{R_2} = P_o + \frac{2\gamma\cos\theta}{R_1} + h\rho g$$

$$h\rho g = 2\gamma\cos\theta\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

$$R_1 = 0.006m$$

$$R_2 = 0.01m$$

$$h = \frac{2\gamma\cos\theta}{\rho g}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = \frac{2\times 0.52\cos 140}{1.36\times 10^4\times 9.8}\left(\frac{1}{0.01} - \frac{1}{0.006}\right)$$

$$= 4.9812\times 10^{-3} m$$

2. A droplet of mercury of radius 2.0mm falls vertically and on hitting the ground it splits into two droplets each of radius 0.50mm. Calculate the change in surface energy. Account for the change in (i) above.

1c) Energy of a large droplet

$= 4\pi r_1^2 \gamma$

$= 4\pi (2 \times 10^{-3})^2 \times 0.52$

$= 2.61 \times 10^{-5}\, J$

Energy of the split drops

$= 2(4\pi r_2^2 \gamma)$

$= 2.(4 \times \pi \times 0.5 \times 10^{-3})^2 \times 0.52)$

$= 3.27 \times 10^{-6}\, J$

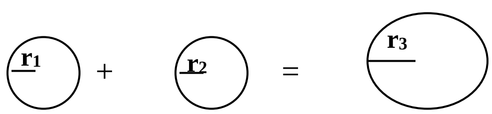
Change in energy

$= 2.61 \times 10^{-5} - 3.27 \times 10^{-6}$

$= 2.283 \times 10^{-5}\, J$

The energy reduces because some of it is lost in overcoming air resistance.

3. Two soap bubbles of radii 2.0cm and 4.0cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is 2.5 x 10^{-2} Nm^{-1}, calculate the excess pressure inside the resulting soap bubble.



$2 \times 4\pi r_1^2 \gamma + 2 \times 4\pi r_2^2 \gamma = 2 \times 4\pi r_3^2 \gamma$

$r_1^2 + r_2^2 = r_3^2$

$r_1 = 0.02m, r_2 = 0.04m$

$r_3 = \sqrt{\ }(0.0004 + 0.0016)$

$r_3 = 0.045m$

$$\text{excess pressure} = \frac{4\gamma}{r_3} = \frac{4 \times 2.5 \times 10^{-2}}{0.045}$$

$$= 2.22 \text{Nm}^{-2}.$$

4. In Jaeger method for measuring the surface of a liquid, the lower end of a capillary tube of radius 0.20mm is 25mm below the surface of the liquid whose surface tension is required and whose density is 8.0 x 10^2 kgm^{-3}. the pressure in the hemispherical bubble

formed at the end of the tube is measured as 40mm on a water manometer. Calculate the surface tension of the liquid.

r = radius of capillary

h = reading on manometer

ρ = density of water

h_1 = height on tube in liquid

ρ_1 = density of specimen liquid.

$$\gamma = \frac{rg}{2}\left(hp - h_1 p_1\right)$$

$$= \frac{0.002 \times 9.8}{2}\left(0.04 \times 100 - 0.0025 \times 8 \times 10^2\right)$$

$$= 9.8 \times 10^{-2} \times 20$$

$$= 1.96 \times 10^{-2}\, Nm^{-1}$$

Exercise

1. Calculate the total pressure inside an air bubble of radius 10^{-5}m at a depth of 0.3 m below the surface of the water.

ii) If the bubble is attached to mercury manometer. Calculate the height to which the mercury rises.

2. A clean glass capillary tube of internal diameter 0.04cm is held with its lower end dipping in water and with 12cm of its tube above the surface.

(i) To what height will water rise in the tube?

(ii) What will happen if the tube is now depressed until only 4cm of its length is above the surface? (surface tension of water is $7.2\text{x}10^{-2}\text{Nm}^{-1}$, angle of contact =0)

3. An oil drop of radius 5cm falls on the ground and breaks into small drops each of radius 2.5cm. Calculate the work done and the speed of the oil drop when it hits the ground. (density of oil is 800 kgm^{-3}; coefficient of surface tension of oil $=1.2 \times 10^{-1}\, Nm^{-1}$

HYDRODYNAMICS / FLUIDS IN MOTION

Streamline / laminar and Turbulent flow

Laminar flow is the orderly flow of the liquid where;

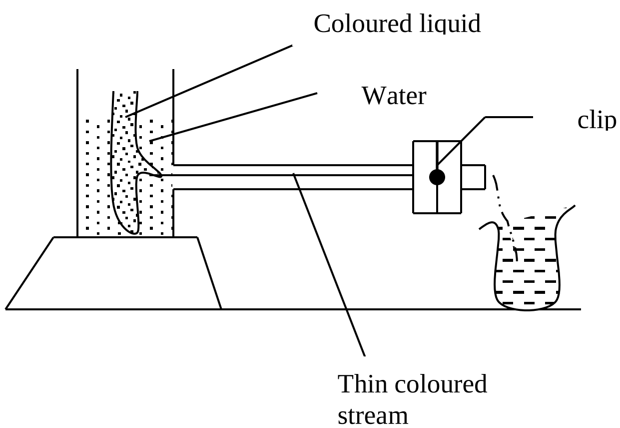
- Lines of liquid flow are parallel to the axis of the tube.
- The particles at the same distance from the axis have the same velocity.
- Laminar flow occurs low liquid velocities.

Turbulent flow

When the flow velocity is increased beyond a critical value (high velocity), wavy currents and sideways movements of the molecules occur and turbulence sets in. the lines of liquid are in random direction.

Experiment to demonstrate laminar and turbulent flow

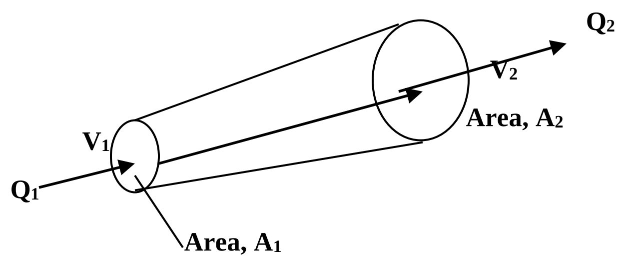
Laminar and turbulent flow cab be demonstrated by introducing a small amount of coloured liquid (Potassium permanganate) at the centre of the tube.



The clip is opened a little to gradually increase the rate of flow. for slow liquid flow rates, a thin coloured stream flow along the axis of the tube showing laminar or orderly flow. For fast rate of liquid flow, the flow of the coloured liquid becomes wavy and spreads out eventually over the whole section of the tube showing turbulent flow.

Continuity Equation

Consider an incompressible liquid (liquid whose density is constant) flowing through a pipe



If a liquid enters a pipe at a rate of Qm^3s^{-1} and leaves at a rate of $Q_2m^3s^{-1}$, then

$Q_1 = Q_2$. this is the continuity equation.

But

$$Q_1 = A_1V$$
$$Q_2 = A_2V_2$$

Hence $\boxed{A_1V_1 - A_2V_2}$ continuity equation

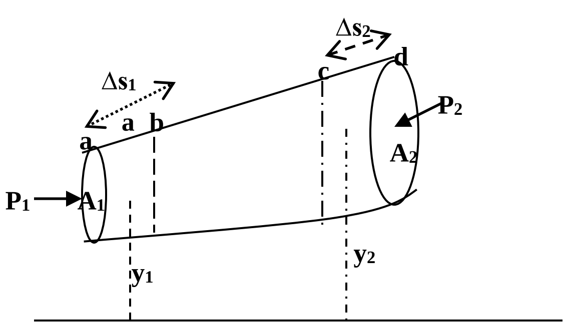
Bernouli's principal

i. An incompressible and non – viscous liquid.

ii. Streamline

iii. Steady state conditions where velocity is independent of time

Consider a section of flow tube, the ends of which have cross sectional areas A_1 and A_2 respectively and are at elevations y_1 and $y_2$above the reference level.



Let p_1 and p_2 be the pressure on the two ends of the flow tube respectively. If v_1 and v_2 are the velocities of the liquid at the inflow and outflow, then in time $\Delta t,$ an amount of liquid $A_1v_1\Delta t$, enters the liquid and an amount $A_1v_1\Delta t$, flows out. By the continuity equation;

$$A_1V_1\Delta t = A_2V_2\Delta t$$
$$v_1\Delta t = \Delta s_1$$
$$V_2\Delta_2 = \Delta s_2$$

$\therefore A_1\Delta s_1 = A_2\Delta s_2$ where Δs_1 and Δs_2 are the displacement of the liquid element between a nad c in time Δt

The force at end a = $F_a = P_1A_1$

And at end d = $F_b = P_2A_2$

The net work done on the element when the liquid element is displaced is

$$\Delta w = P_1A_1\Delta s_1 - P_2A_2\Delta s_2$$

But

$$A_1\Delta s_1 = A_2\Delta s_2 = \Delta v$$

where Δv = change in volume

$$\Delta w = p_1\Delta v - p_2\Delta v$$
$$= (p_1 - p_2)\Delta v \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The mass of the liquid having volume Δv is $\rho\Delta v$

The kinetic energy of the mass of the liquid entering at a is $\frac{1}{2}mv^2 = \frac{1}{2}(p\Delta V)v_1^2$ and that of the liquid leaving at d is $\frac{1}{2}(p\Delta V)v_2^2$

The net change in K.E = $\frac{1}{2}(P\Delta V)v_2^2 - \frac{1}{2}(P\Delta V)v_1^2 = \frac{1}{2}(P\Delta V)(v_2^2 - v_1^2)$

The net gain in gravitational potential energy = $p(\Delta v)gy_2 - p(\Delta v)gy_1 = p\Delta vg(y_2 - y_1)$

The change in mechanical energy = $K.E + P.E = \frac{1}{2}(p\Delta v)(v_2^2 - v_1^2) + p\Delta vg(y_2 - y_1)$

The work done on the liquid element = the change in mechanical energy i.e

$$(p_1 - p_2)\Delta v = \frac{1}{2}p\Delta V(v_2^2 - v_1^2) + \rho\Delta Vg(y_2 - y_1)$$
$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

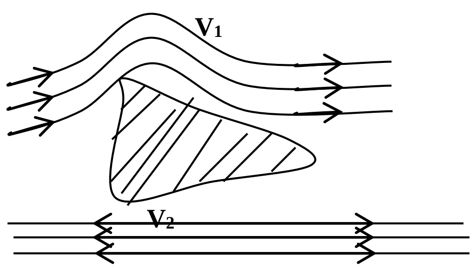
$p_1 + \frac{1}{2}pv_1^2 + \rho gy_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gy_{2.}$ This is Bemouli's equation

Hence

$$p + \frac{1}{2} pv^2 + pgy = \text{constant.}$$

For an incompressible non viscous liquid, the sum of pressure kinetic energy per unit volume, potential energy per unit volume is constant for laminar flow.

Applications of Bernouli's principle



The orientation of aerofoil relative to the flow direction cause the flow lines to crowd together above the aerofoil corresponding to increased flow velocity. And according to Bernoulli's equation the pressure above reduces. Below the aerofoil, the flow velocity is lower and hence the pressure is higher, hence there is a resultant thrust upwards leading to the lift.

Jets and nozzles

Bemouli's equation suggests that for fluid flow where potential energy change is very small or zero as in a horizontal pipe, the pressure falls when the velocity rises. The velocity increases at constriction.

The greater the change in cross-sectional area, the greater is the increase of velocity and so the greater is the pressure drop.

$A_1V = A_2V_2$ $\qquad$ $\mathrm{A}_1 > A_2$ $\qquad$ $\mathrm{V}_2 > V_1$

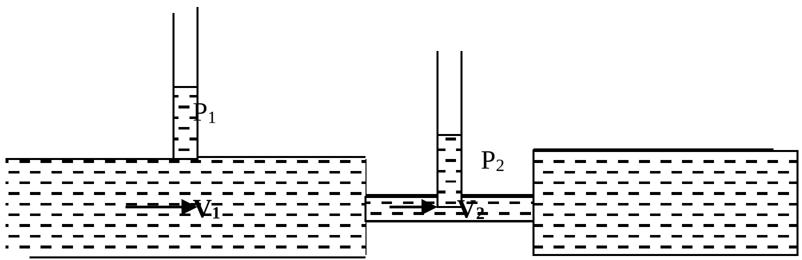
Several devices with jets and nozzles use this effect e.g Bunsen burner, filter pump and paint spray.

iii) Flow meters

These measure the rate of flow of a fluid through a pipe.

a) Venturi flow meter

This consists of a horizontal tube with a constriction and replaces part of a piping of a system.



The two vertical tubes record the pressures in the fluids flowing in the normal part of the tube and in the constriction.

From Bernoulli's equation (ρgy is not considered because pipe and constriction are at the same level)

$$p_1 + \frac{1}{2}\rho v_1{}^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$p_1 - p_2 = \frac{1}{2}pv_2^2 - \frac{1}{2}pv_1^2$$

Using the equation of continuity

$$A_1V_1 = A_2V_2$$

$$V_2 = \frac{A_1V_1}{A_2}$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(\frac{A_1^2v_1^2}{A_2^2} - v_1^2\right)$$

$$\left(P_1 - P_2\right) = \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)v_1^2$$

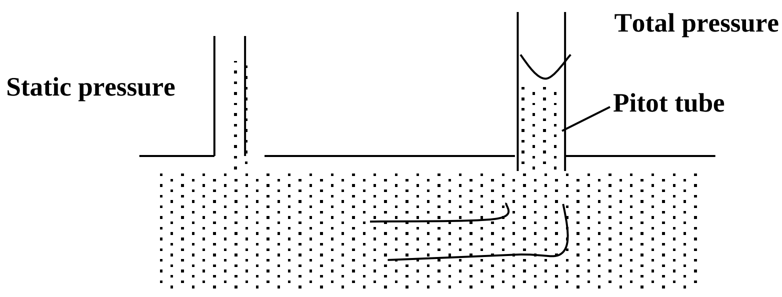
b) Pitot tube

The pressure exerted by a moving fluid called total pressure can be regarded as having two components namely;

i. The static pressure which it would have if it were to rest .

ii. Dynamic pressure which is the pressure equivalent of its velocity $\left(\frac{1}{2}\rho v^2\right)$

A pitot tube measures total pressure.



Total pressure = static pressure + dynamic pressure

Dynamic pressure = total pressure – static pressure

$$\frac{1}{2}\rho v^2 = (\text{Total pressure - static pressure})$$

$$\text{v}^2 = \frac{2}{\rho}(\text{total pressure - static pressure})$$

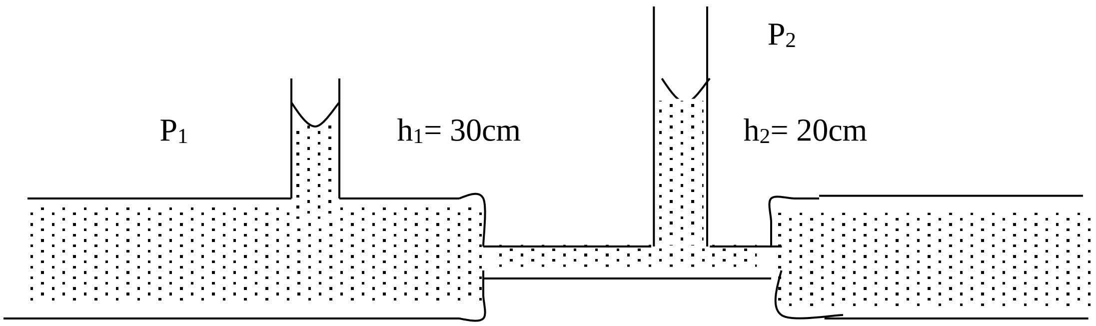
Questions

1. At a certain section of the horizontal water pipe, the static pressure is 1.96 x 10^5Pa, the total pressure is 2.04 x 10^5Pa and area of cross section is 20cm^2, if the density of water is 10^3kgm^{-3}, find the volume flow rate in the pipe.

Solution :

$$v^2 = \frac{2}{\partial}(\text{total pressure - static pressure}).$$

$$v^2 = \frac{2}{10^3}\left(2.04\times10^3 - 1.96\times10^5\right)$$
$$= 0.002((8000)$$
$$v = 4ms^{-1}$$
$$\therefore \text{flow rate} = \text{AV}$$
$$= \frac{20}{10{,}000}\times 4$$
$$= 8\times10^{-4}m^3s^{-1}$$



2. The above diagram represents a venture-meter, if the cross sectional area of the main pine is 5.81 x 10^{-3}m^2 and that of the constriction is 2.58 x 10^{-3}m^2, find the velocity v

Solution

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$
$$p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$
but $\text{A}_1V_1 = A_2V_2$
$$V_2 = \frac{A_1V_1}{A_2}$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(\frac{A_1^2V^2}{A_2^2} - V_1^2\right)$$
$$P_1 - P_2 = \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)V_1^2$$

$$P_1 = P_0 + h_1 \rho g \qquad\qquad \mathrm{P}_2 = P_0 + h_2 \rho g$$

$$(h_1 - h_2)\rho g = \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)V^2$$

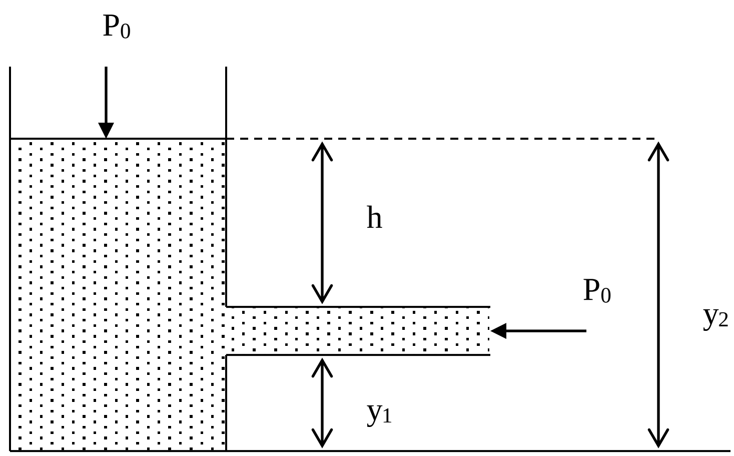
$$V_1^2 = \frac{(h_1 - h_2)}{\frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)}$$

$$V_1^2 = \frac{98}{\frac{1}{2} \times 4.07} \qquad = \frac{98}{2.04}$$

$$V_1^2 = 48.14$$

$$V_1 = 6.9ms^{-1}$$

<u>Flow velocity of a liquid from a tank open to the atmosphere.</u>



By Bemouli's principal,

$$p_o + \rho g y_2 = P_0 + \rho g y_1 + \frac{1}{2}\rho v^2$$

Where v is the velocity from the orifice near the bottom of the tank

$$\rho g(y_2 - y_1) = \frac{1}{2}\rho v^2$$

but $\mathrm{y}_2 - y_1 = h$

$$\rho g y = \frac{1}{2}\rho v^2$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

VISCOSITY

This is the resistance between fluid layers in contact moving relative to each other.

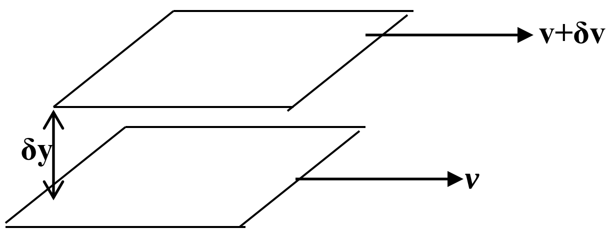
If adjacent layers of a material are displaced laterally over each other, the deformation of the material is called shear.

All liquids and gasses stick to a solid surface so that when they flow, the velocity must gradually decrease to zero as the wall of the pipe is approached, a fluid is therefore sheared when it flows past the solid surface. The opposition set up by the fluid to shear is called the viscosity. So viscosity is a kind of internal friction exhibited to some degree by all fluids.

It arises in liquids because the forced movement of a molecule relative to its neighbours is opposed by the intermolecular forces between them. But viscosity of a liquid is the measure of its resistance to flow. The greater the viscosity, the less easier it is for a liquid to flow and the more sticky it is hence oil is said to be more viscous than water.

Coefficient viscosity, η

Consider two parallel layers of liquid separated by distance δy and having velocities v + δv



The frictional force F between the layers F shear stress = $\frac{F}{A}$ Where A is the area of the layers.

The rate of change of shear strain is ${dv}/{dy}$, this is also called strain rate or velocity gradient.

For lamina flow

$$\frac{F}{A} \propto \frac{\partial v}{\partial y}$$

$$\frac{F}{A} = \eta \frac{\partial v}{\partial y}$$

$$\eta = \frac{F}{\left(\frac{A\partial v}{\partial y}\right)} = \text{coeffient of viscosity}$$

$$\eta = \frac{\text{shear stress}}{\text{shear strain}}$$

Coefficient of viscosity is the stress which results the motion of one layer of a fluid over another when the velocity gradient is unit or it is the frictional force per unit area when its in a region of unit velocity gradient.

Unit of η is $Nm^{-2}s$ or Pas

Question: Prove that [η] = $ML^{-1}T^{-1}$

Poiseulle's equation (For lamina flow only)

The volume rate of flow of a liquid through a pipe depends on;

i. The radius *r* of the pipe
ii. The coefficient of viscosity η
iii. The pressure gradient $\left(\frac{P}{l}\right)$ where P is the pressure head and *l* is the length of the tube.

$$\left(\frac{v}{t}\right) = kr^{x}\eta^{y}\left(\frac{p}{l}\right)^{z}$$

$$[L.H.S] = \frac{[V]}{[t]} = \frac{L^3}{t} = L^3T^{-1}$$

$$[R.H.S] = K[r]^{x}[\eta]^{y}\left[\frac{p}{l}\right]^{z}$$

$$= KL^{X}(ML^{-1}T^{-1})^{y}\left(\frac{ML^{-1}T^{-2}}{L}\right)^{z}$$

$$= KL^{X}M^{y}L^{-Y}M^{z}L^{2Z}T^{-2Z}$$

$$= KM^{y+z}L^{x-2z-y}T^{-Y-2Z}$$

resolving Left hand side and Right hand side

$M; y + z = 0$....................(1)

$L; x\text{-}2z\text{-}y = 3$....................(2)

$T; y - 2z = ^{-}1$....................(3)

From equation(3) ; y + 2z = 1

y = 1 – 2z

Put in equation (1);1-2z + z = 0

1-z = 0

Z = 1

$\therefore y = -1$

Using equation (2)

$x - 2 + 1 = 3$

$x = 4$

$\therefore$ poiseulle's equation is

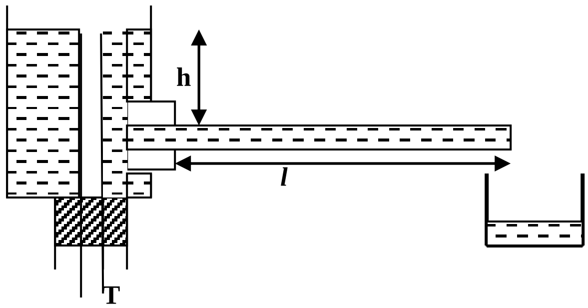
$$\therefore \frac{v}{t} = k\eta^{-1}r^{4}\left(\frac{p}{t}\right)$$

$$\frac{v}{t} = \frac{kr^{4}p}{\eta l}$$

$$but k = \frac{\pi}{8}$$

$\frac{v}{t} = \frac{\pi}{8}\frac{pr^{4}}{\eta l} \Rightarrow$ Poiseulle's equation (only for laminar flow)

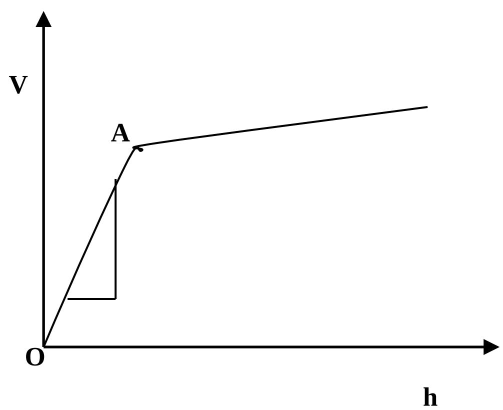
Determining coefficient of viscosity using Poiseulle's equation (Apply only to less viscous liquids e.g. water



The pressure head h is varied by raising or lowering tube T

Liquid flowing through the capillary tube is collected for a measured time. The volume of water, V, flowing per second is calculated.

A graph of V against h is plotted;



The scope, S of the graph an region OA is determined from

$$\frac{V}{t} = \frac{\pi}{8}\frac{pr^4}{\eta l}$$

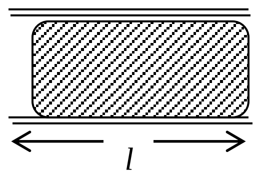
But p = hρg

$$\frac{V}{t} = \frac{\pi}{8}\frac{h\rho g r^4}{\eta l} = \left(\frac{\pi \rho g r^4}{8\eta l}\right)h$$

$$\text{The slope, S} = \left(\frac{\pi \rho \text{gr}^4}{8\eta \text{l}}\right)$$

$$\eta = \frac{\pi \rho g r^4}{8Sl}$$

In determining the radius of the tube, mercury of known mass is filled in the tube



$\pi r^2 l p g h = m$

Stoke's law

A body moving in a fluid experiences a retarding force due to the viscosity of the fluid. This retarding force is called viscous drag.

Note. The difference between viscosity and viscous drag is that viscosity is a frictional force which opposes relative motion between liquid layers whereas viscous drag is a frictional force experienced by a body in a fluid.

The viscous drag F, experienced by a sphere moving in a fluid depend on

i. The radius r of the sphere
ii. The velocity v of the sphere.
iii. The coefficient of viscosity η of the liquid.

For a constant body of similar dimensions moving in a uniform fluid, the force of viscosity depends on the velocity of the body.

Hence $F = kr^x \eta^y v^z$

$MLT^{-2} = [L]^x (ML^{-1}T^{-1})^y (LT^{-1})^z$

$= L^x.M^y.L^{-y}.T^{-y}.L^Z.t^{-Z}$

$MLT^{-2} = L^{x-y+z}.M^y T^{-y-z}$

$M^1 = M^y$, hence $y = 1$

$L^1 = L^{x-y+z}$, hence $x - y + z = 1$

but $y = 1, x + z = 2$

$T^{-2} = T^{-y-z}, hence - y - z = -2$

$but y = 1, z = 1$

$hence x = 1$

$\therefore F = kr\eta v$

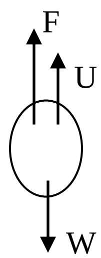
Detailed analysis indicate that

$k = 6\pi$

$\therefore F = 6\pi r\eta v \rightarrow$ Stoke's law

Motion of a metal sphere in a viscous liquid

Consider the forces acting on the sphere as it falls through a liquid



W = weight

U = upthrust

F = viscous drag

The resu ltant force on the sphere is W- (F+U)

From Newton’s second law; $\frac{mdv}{dt} = w - (F + U)$ where m = mass of the sphere

If a is th radius of the sphere, ρ the density of the material of the sphere and σ the density of the liquid then

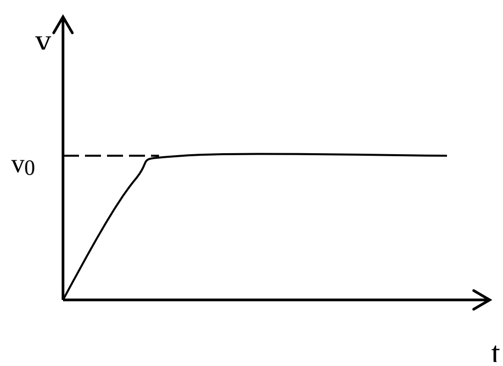
$$W = vpg = \frac{4}{3}\pi a^3 pg$$
$$U = \frac{4}{3}\pi a^3 \sigma g$$
$$F = 6\pi a \eta v$$

The sphere will accelerate until the net force on it is zero, hence W – (F+U) = 0

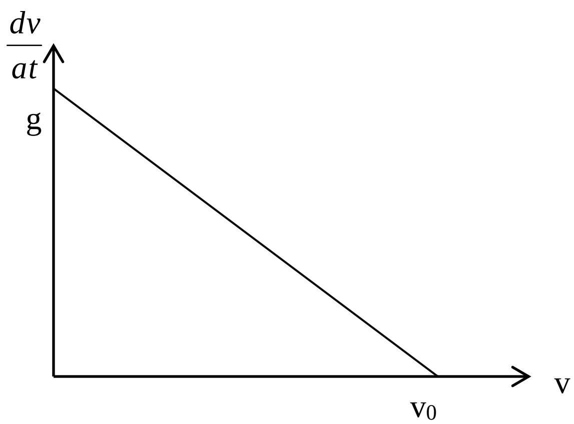
When the net force on the sphere is zero, it moves with a constant velocity V_o called ***terminal velocity***.

$$W = F + U$$
$$\frac{4}{3}\pi a^3 pg = \frac{4}{3}\pi a^3 \sigma g + 6\pi a \eta v_0$$
$$v_0 = \frac{4}{3}\frac{\pi a^3 g}{6\pi a \eta}(p - \sigma)$$
$$= \frac{2}{9}\frac{a^2 g}{\eta}(p - \sigma)$$
$$\therefore v_0 = \frac{2a^2}{9\eta}(p - \sigma)g$$

A sketch of velocity against time for a sphere moving in a viscous liquid.



A graph of acceleration against velocity.



Measurement of coefficient of viscosity using Stoke's Law

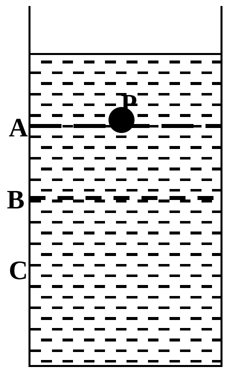
From the equation

$$v_0 = \frac{2a^2}{9\eta}(p-\sigma)g$$

$$\eta = \frac{2a^2}{9v_0}(p-\sigma)g$$

The method is suitable for very viscous liquids such as oil.

The densities ρ and σ of the material of the sphere and the specimen liquid respectively are determined.



A tall glass tube T supported vertically in a constant temperature enclosure. Three reference marks ABC are made along the tube T using rubber bands at equal spaces.

A ball bearing is moistened with a specimen liquid and then allowed to fall vertically down a liquid by releasing it. The times t_1 taken by the ball to fall from A to B or t_2 from B to C are measured. The equality of these two times implies that the sphere had attained terminal velocity by the time it reached point A. if t_1 is not equal to t_2 the reference marks are drifted further down the tube and the repeated.

If $t_1 = t_2 = t$, then terminal velocity

$$v_0 = \frac{AB}{t}$$

$$From v_0 = \frac{2a^2}{9\eta}(p-\sigma)g$$

The η can be calculated.

When the experiment is repeated with a liquid of coefficient of viscosity η_1 and density σ_1, using the same ball-bearing, then.

$$\eta_1 = \frac{2a^2}{9v_1}(p - \sigma_1)$$

Where v_1 is the new terminal velocity .

$\therefore \frac{\eta}{\eta_1} = \frac{v_1(\rho-\sigma)}{v_0(\rho-\sigma_1)}$ Thus knowing v_1,v,ρ,σ_1,σ,the coefficients of viscosity can be compared.

Effect of temperature on viscosity of fluids

Liquids

The viscosity of a liquid decreases as the temperature rises. When the temperature increases, the molecules of the liquid on the average are further apart and the intermolecular attractive forces decrease.

The resistance to flow decreases hence coefficient of viscosity decreases.

Gases

Viscosity in gases is due to molecules in gases moving from the slower moving layers to the fast moving layers and from the fast moving layers to the slow moving layers. The net result of this is more momentum is carried one way than the other. This is turn means that forces exist on the layers which retard the fast moving layers and accelerate the slower moving layers. The retardation depends on the mass of the molecules and their speeds i.e. the momentum, mv.

When the temperature of the gas is raised, the molecular speeds and hence the momentum increase, leading to an increase in the viscosity of the gas.

Examples

1. A flat plate of area $0.1m^2$ is placed on a flat surface and is separated from the surface by a film of oil 10^{-5}m thick, where η is $1.5Nsm^2$. Calculate the force required to cause the plate to slide on the surface at a constant speed of $1mms^{-1}$.

$$\eta = \frac{\text{shear stress}}{\text{strain rate}} = \frac{F/A}{dv/dy}, 1.5 \Rightarrow \frac{F}{0.1} \times \frac{10^{-5}}{10^{-3}} \quad , \quad \mathrm{F} = \frac{1.5 \times 0.1}{10^{-2}} \quad = 15\mathrm{N}$$

2. The terminal velocity of a spherical oil drop falling in air at 20^0c is $2 \times 10^{-7}ms^{-1}$. What is the radius of the drop if its density is $930kgm^{-3}$?

   Assume η of air at 20^0c = 1.8×10^{-5}Pas

   Density of air = $1.2kgm^{-3}$

$$V_0 = \frac{2a^2}{9\eta}(p-\sigma)g$$

$$2\times10^{-7} = \frac{2a^2}{9\times1.81\times10^{-5}}(930-1.2)9.8$$

$$3.258\times10^{-11} = 18204.48a^2$$

$$a = 4.2\times10^{-8}m$$

3. A steel ball bearing of diameter 8.0mm is timed as it falls through oil at a steady speed. Over a vertical distance of 0.20m, it takes 0.56s. Assuming the density of steel is 7.8×10^{-3} kgm^{-3} and that of oil 9.0×10^2 kgm^{-3}. Calculate;
   a) Weight of the ball
   b) Upthrust on the ball
   c) Viscosity of the oil

Using stokes law

$$V = \frac{2a^2}{9\eta}(p-\sigma)g$$

$$\text{given a} = 4\times10^{-3} \qquad \text{V} = \frac{0.2}{0.56} \qquad = 0.36$$

$$\text{P} = 7.8\times10^3$$

$$\sigma = 9\times10^2 kgm^{-3}$$

Weight

$$= \tfrac{4}{3}\pi a^3 pg$$

$$= \tfrac{4}{3}\pi(4\times10^{-3})^2 \times7.8\times10^3\times9.8 = 0.2N$$

Upthrust

$$= \frac{4}{3}\pi r^3 pg$$
$$= \frac{4}{3}\pi\left(4\times10^{-3}\right)^3 \times 9\times10^2 \times 9.8$$
$$= 0.0024N$$

Viscosity of oil

$$\eta = \frac{2a^2}{9v_0}\left(p-\sigma\right)g$$
$$= \frac{2\times\left(4\times10^{-3}\right)^2}{9\times0.36}\times9.8\left(7.8\times10^3 - 9\times10^2\right)$$
$$= 0.6679 Pas$$

4. A spherical raindrop of radius 2 x 10^{-4} m falls vertically in air at 20^0c. If the densities of air and water are 1.2kgm^{-3} and 1000kgm^{-3} and the viscosity of air 20^0c is 1.8 x 10^{-5} Pas. Calculate the terminal velocity of the drop.

$$V_0 = \frac{2a^2}{9\eta}\left(p-\sigma\right)g$$
$$= \frac{2\times\left(2\times10^{-4}\right)^2}{9\times1.8\times10^{-5}}\left(1000-1.2\right)9.8$$
$$= 4.81 ms^{-1}$$

Exercise

1. Air flows past the upper surface of a horizontal aero plane wing at 250ms^{-1} and past the lower surface of the wing at 200ms^{-1}. The density of air is 1.0kgm^{-3} at the flight altitudes and the area of the wing is 20m^2. Calculate the net lift on the wing. (2.25x10^5N)

2. A pitot – static force fitted on a pressure gauge is used to measure the speed of a boast at sea. Given that the speed of the boat does not exceed 10ms^{-1} and the density of sea water is 1050 kgm^{-3}, calculate the maximum pressure on the gauge. (5.25x10^4Pa)

ELASTICITY

Mechanical properties of materials.

The following are used to describe different mechanical characteristics of materials:

Strength: It is the ability of the material to withstand an applied force before the material breaks.

Stiffness: This is the resistance which a material offers to having its shape or size changed.

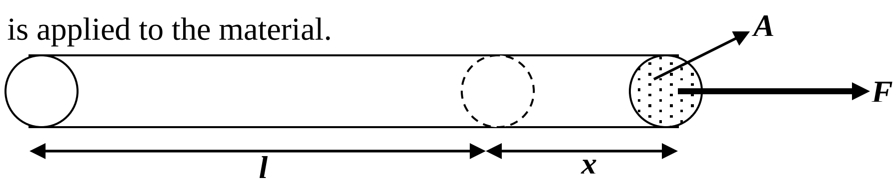
Ductility: This is the ability of a material to be hammered, bent, rolled, or pressed into different shapes. Ductile materials undergo both elastic and plastic deformation.

A material is said to undergo elastic deformation when it can regain its original length after the stretching forces are removed. A material undergoes plastic deformation when it does not regain its original length when the stretching forces are removed.

Brittle material cannot be permanently stretched. It undergoes elastic deformation not plastic deformation.

Tensile Stress, Tensile Strain and Young's Modulus.

Suppose a material of length *l*, cross section, *A*, stretched by an extension *x* when a force F is applied to the material.



Tensile stress is the ratio of the force to the cross section area.i.e.

$$\text{Tensile stress} = \frac{force}{area} = \frac{F}{A}$$

Unit of stress is Nm^{-2} or Pascals (Pa).

Dimensions of stress = [stress] = $ML^{-1}T^{-2}$.

Tensile strain is the ratio of the extension to the original length of the material. i.e.

$$\text{tensile strain} = \frac{extension}{original\,length} = \frac{x}{l}$$

Strain has no units.

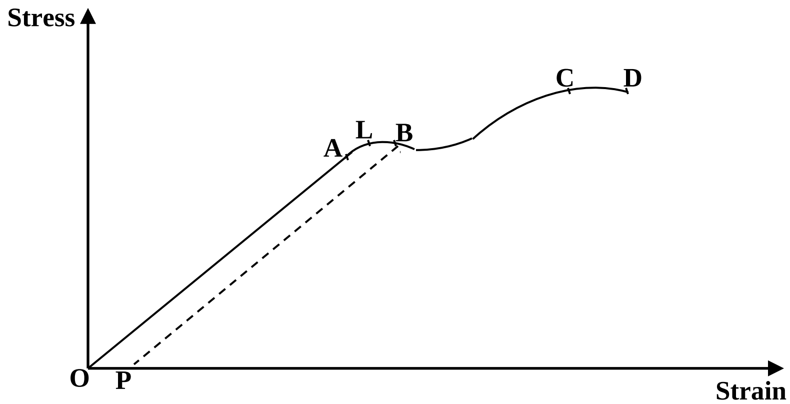
Young's Modulus, E.

This is the ratio of tensile stress to tensile strain.

$$\text{Young's Modulus, E} = \frac{Stress}{Strain} = \frac{\frac{F}{A}}{\frac{x}{l}} = \frac{Fl}{Ax}.$$

$$\text{Therefore, stretching force, } F = \frac{EAx}{l}.$$

A graph of Stress against strain for a ductile material



OA is a straight line. Up to point A stress is proportional to strain. The portion OA is the Hooke's law region. Region OA is where young's modulus is defined. A is called the *proportional limit*. Along OA and up to L just beyond A, the wire returns to its original length when stress is decreased to zero. L is called *elastic limit*. Beyond L up to B the material becomes plastic. The molecules of the wire begin to slide across each other and some of the energy of the material is dissipated as heat. Point B is the *yield point*. With further increase in stress, work hardening occurs; this is due to the dislocations. When the dislocation density is high slippage of atomic plates became difficult. The dislocations become tangled up with each other. Point C is the *breaking stress or maximum stress*. At this point the material develops kinks. Point D is the where the wire breaks.

Question: Sketch graphs on the same axes of stress against strain for glass, metal wire and rubber, and explain the nature of the graphs.

Force in a metal bar due to contraction or expansion

When a bar is heated, and then prevented from contracting as it cools, a considerable force is exerted at the ends of the bar. Consider a bar of young's modulus, E, a cross sectional A, linear expansivity α, and a decrease in temperature ΔΘ°C. If *l* is the original length of the bar, the decrease in length x if the bar were free to contract = α*l* (ΔΘ). Now

$F = \frac{EAx}{l}$,   but x = α*l* (ΔΘ)

$$F = \frac{EA(\alpha l \Delta\theta)}{l} = EA\alpha(\Delta\theta)$$

Relationship between Young's modulus, E and the force constant, k

From the definition of young's modulus, $E = \frac{Fl}{Ax}, F = \left(\frac{EA}{l}\right)x$ ………(i)

Using Hooke's law, F = kx …………………………………………….(ii)

From equations (i) and (ii) $k = \frac{EA}{l}$

Energy stored in a stretching wire

Suppose a wire is stretched by an amount x by applying a force F without exceeding elastic limit. The average force = (0+F)/2 = ½F.

Now the work done = force x distance.

Work done = average force x extension= ½F. *x*

.This is the amount of energy stored in the wire.

Further, since $F = \frac{EAx}{l}$,

energy stored = $\frac{EAx^2}{2l}$.

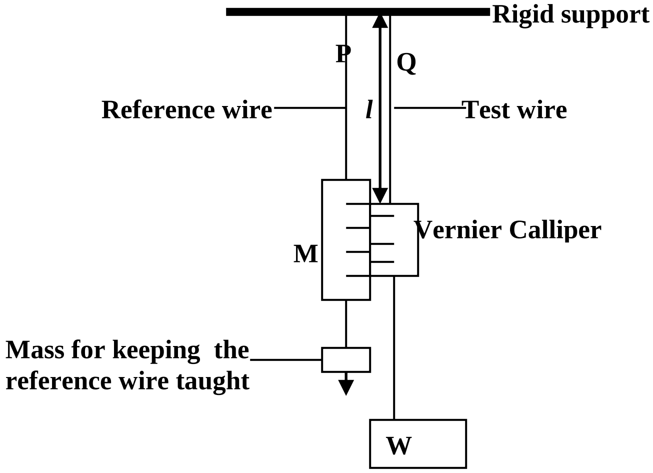
Energy stored per unit volume

energy stored = $\frac{EAx^2}{2l}$ but volume = A*l*.

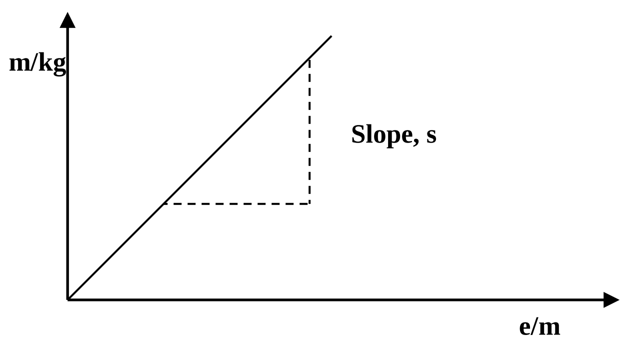
energy stored per unit volume = $\frac{EAx^2}{2l.Al} = \frac{E}{2}\left(\frac{x}{l}\right)^2 = \frac{Young's\ modulus \times}{2}(strain)^2$

$= \frac{Stress}{2strain} \times (strain)^2$ = ½ x stress x strain

Experiment to determine Young's Modulus for a metal wire



Two thin, long wires of the same material and length P and Q are suspended from a rigid support. P carries a scale M in mm and its straightened by attaching a weight at its end. Q carries a vernier scale which is along side scale M. Various loads are added to the test wire and the corresponding extensions caused are read off from the vernier scale. After each reading, the load should be removed to check that the wire returns to its original position, showing that elastic limit has not been exceeded. The original length of the wire l is measured from the rigid support up to the vernier scale. Using a micrometer screw gauge, the diameter of the test wire and hence the cross sectional wire $A = \pi r^2$ can be obtained. A graph of mass(m) of the load against extension(e) is plotted.



From $E = \dfrac{mgl}{eA}$, $\quad m = \dfrac{EA}{gl} e$

Slope, $s = \frac{EA}{gl}$

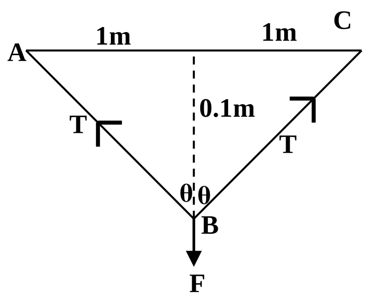
Hence $E = \frac{gsl}{A}$

Examples:

1. A metal wire of diameter $2.0x10^{-4}$m and length 2m is fixed horizontally between two points 2m apart. Young's modulus for the wire is $2x10^{11}Nm^{-2}$.

(i) What force should be applied at the mid point of the wire to depress it by 0.1m.

(ii) Find the work done in (i) above.



1.

$\cos\theta = \frac{0.1}{AB}$ but $AB = \sqrt{(1^2 + 0.1^2)} = 1.005m$

hence $\cos\theta = \frac{0.1}{1.005}$

ABC = 2xAB = 2x1.005 = 2.01m, Extension, e = 2.01 - 2 =0.01m

$T = \frac{EAe}{l}$ and A = $\pi r^2 = \pi d^2/4$

Resolving vertically, 2Tcosθ = F,

Therefore, $F = \frac{2EAe\cos\theta}{l} = \frac{2E\pi d^2 e\cos\theta}{4l}$

$$F = \frac{2\times 2\times 10^{11}\times\pi\times(2\times 10^{-4})^2\times 0.01\times 0.1}{1\times 4\times 1.005} = 12.5N$$

ii) Work done = ½Fe = ½x12.5x0.01 = 0.0625J

2. A uniform metal bar of length 1.0m and of diameter 2.0cm is fixed between two rigid supports at 25°C. If the temperature of the rod of raised to 75°C. Find (i) the force exerted on the supports. (ii) The energy stored in the rod at 75°C (Young's modulus for the metal = 2.0×10^{11}Pa, coefficient of linear expansion = $1.0 \times 10^{-5}K^{-1}$)

(i) $F = EA\alpha(\Delta\theta)$

$F = 2.0\times10^{11}\times(\pi\times0.01^{2})\times1.0\times10^{-5}(75-25) = 31400N$

(ii) Energy stored = ½Fe, but e =$\alpha l(\Delta\theta)$

Hence energy stored = ½$F\alpha l(\Delta\theta)$ = ½x31400x1.0x10^{-5}x1x(75-25) =7.85J

Exercise

1. A thin steel wire initially 1.5m long and of diameter 0.5mm is suspended from a rigid support. Calculate (i) the final extension, (3.53×10^{-3}m)
(ii) the energy stored in the wire, when a mass of 3kg is attached to the lower end. (Young's modulus of steel = $2.0 \times 10^{11}Nm^{-2}$) ($5.19 \times 10^{-2}$J)

2. Two thin wires, one of steel and the other of bronze each 1.5m long and of diameter 0.2cm are joined end to end to form a composite wire of length 3m. What tension in this wire will produce a total extension of 0.064cm? (Young's modulus for steel = 2×10^{11}Pa, Young's modulus for bronze = 1.2×10^{11}Pa) (1009N)

3. A copper wire and steel wire each of length 1.0m and diameter 1.0mm are joined end to end to form a composite wire 2.0m long. Find the strain in each wire when the composite stretches by 1.0×10^{-3}m. (Young's moduli for copper and steel are 1.2×10^{11}Pa and 2.0×10^{11}Pa respectively).

4. The ends of a uniform wire of length 2.00m are fixed to points A and B are 2.00m apart in the same horizontal line. When a 5kg mass is attached to the mid-point C of the wire, the equilibrium position of C is 7.5cm below the line AB. Given that young's modulus for the material of the wire is 2.0×10^{11}Pa, find:
(i) the strain in the wire,
(ii) the stress in the wire,
(iii) the energy stored in the wire.

Answers

Exercise 1:

a) Force $= \frac{Massxlength}{(time)^2}$

(b) Pressure= $\frac{Mass}{lengthx(time)^2}$

(c) work $= \frac{forcex(length)^2}{(time)^2}$

(d) momentum = $\frac{massxlength}{time}$

Exercise 2:

(a) [Density] = ML^{-3}

(b)[Pressure] = $ML^{-1}T^{-2}$

(c) [Power] = $ML^{2}T^{-3}$

(d)[Momentum] = MLT^{-1}

Exercise 3:

Velocity ratio, logarithmic numbers, efficiency, coefficient of friction,

Exercise 4:

1. x = 1, y = 1 and z = 1
2. x = 1, y = 1 and z = 1

Exercise 5:

1. 6.06N

2. 13.3ms^{-2}

Exercise 6:

1. $V = gt$, $S = ½gt^2$, $V^2 = 2gs$

2. $t = U/g$, $s = U^2/2g$

Exercise 7

2. (i) 9.9s, (ii) 27.7ms^{-1} (iii) 137.2m

Exercise 8:

1. 83.1m, 2. angle of projection 53.1°, initial speed = 63.9ms^{-1}

Exercise 9:

1. (i) 3004N (ii) 1114.3N
2. (i) 4.905ms^{-2} (ii) 98.1N (iii) 58.87J